AM-165

Problem Set 4

Sept. 2001

3.3 Similar to Exercise 2.119. The event $Y = 2$ occurs if the first and second components tested are both defective.

$p(2) = P(DD) = \frac{2}{4}\left(\frac{1}{3}\right) = \frac{1}{6}$

$p(3) = P(DGD) + P(GDD) = 2\left(\frac{2}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right) = \frac{2}{6}$

$p(4) = P(GGDD) + P(DGGD) + P(GDGD) = 3\left(\frac{2}{4}\right)\left(\frac{1}{3}\right)\left(\frac{2}{2}\right) = \frac{1}{2}$

Since there are only four components, $Y = 2, 3$, and 4 are the only possible values for the random variable Y.

3.4 Define the following events:

A: valve 1 fails $\quad$ B: valve 2 fails $\quad$ C: valve 3 fails

Notice :

$P(Y=2) = P(\overline{A} \cap \overline{B} \cap \overline{C}) = .8^3 = .512,$

$P(Y=0) = P(A \cap (B \cup C)) = P(A)P(B \cup C) = .2(.2 + .2 - (.2)^2) = .072,$

(by the law of total probability) $P(Y=1) = 1 - .512 - .072 = .416.$

3.25 $B = SS \cup FS$

$P(B) = P(SS) + P(FS)$ $\qquad$ SS and SF are mutually exclusive

$= \frac{2000}{5000} \times \frac{1999}{4999} + \frac{3000}{5000} \times \frac{2000}{4999}$

$= \frac{2000}{5000}\left(\frac{1999}{4999} + \frac{3000}{4999}\right) = \frac{2000}{5000} = 0.4.$

$P(B|\text{first trial success}) = \frac{1999}{4999} = 0.3999$

which is <u>not</u> markedly different from 0.4.

3.29 Let Y be the number answered correctly. Then $p = P(\text{correct answer}) = \frac{1}{5}$ and $n = 15$.

$P(Y \geq 10) = 1 - P(Y \leq 9) = 1 - 1.000 = .000$ $\qquad$ (to three decimal places)

using Table 1, Appendix III.

By explicit numeric calculation, the answer is

3.31 Let Y be the number of qualifying subscribers. Then Y has a binomial distribution with $p = .7$ and $n = 5$. Though we could easily use table 1 (appendix III) we perform the calculations exactly.

a. $P(Y=5) = \binom{5}{5}(.7)^5 = .1681$

b. $P(Y \geq 4) = P(Y=4) + P(Y=5)$

$= \binom{5}{4}(.7)^4(.3) + \binom{5}{5}(.7)^5$

$= .3601 + .1681 = .5282$

3.37 Let Y be the number of housewives preferring brand A. Under the assumption that there is no difference between brands, $p = P(\text{prefer brand } A) = .5$ and $n = 15$.

a. Using Table 1, Appendix III,

$$P(Y \geq 10) = 1 - P(Y \leq 9) = 1 - .849 = .151$$

b. $P(10 \text{ or more prefer } A \text{ or } B) = P(Y \leq 5 \text{ or } Y \geq 10) = P(Y \leq 5) + [1 - P(Y \leq 9)]$ $= .151 + (1 - .849) = .302$, since 10 or more preferring B is equivalent to 5 or less preferring A.

3.46 **a.** Independence of the three inspection events.

b. Let Y = # of planes with wing cracks that are detected. Y has a binomial distribution with $n = 3$ and $p = (.9)(.8)(.5) = .36$. $P(Y \geq 1) = 1 - P(Y = 0)$ $= 1 - \binom{3}{0}(.36)^0(.64)^3 = .737856$

3.54 Let Y be the number of holes drilled until the first procedure well is found. Then Y has a geometric distribution with $p = .2$.

a. $p(3) = (.8)^2(.2) = .128$

b. $P(Y > 10) = P(\text{first 10 holes are nonproductive}) = (.8)^{10} = .107$

3.55 **a.** $P(Y > a) = \sum_{y=a+1}^{\infty} q^{y-1}p = q^a \sum_{y=a+1}^{\infty} q^{y-a-1}p = q^a \sum_{z=1}^{\infty} q^{z-1}p = q^a.$

(Notice $\sum_{z=1}^{\infty} q^{z-1}p = 1$ by problem 3.44)

b. Using the result of part **a**,

$$P(Y > a + b | Y > a) = \frac{P(Y>a+b, Y>a)}{P(Y>a)} = \frac{q^{a+b}}{q^a} = q^b = P(Y > b)$$

Let Y represent the time (in years) until failure of an electrical component. Then **b.** suggest that the probability the component last b or more years is q^b regardless of how long the component has already lasted. That is, the life of the component has no memory of the past.

3.61 Define Y to be the number of people questioned before a "yes" answer is given. Then

$p = P(\text{yes}) = P(\text{smoker} \cap \text{yes}) + P(\text{nonsmoker} \cap \text{yes}) = P(\text{yes}|\text{smoker})P(\text{smoker}) + 0$ $= .3(.2) = .06.$

Thus,

$$p(y) = pq^{y-1} = .06(.94)^{y-1}, \qquad y = 1, 2, 3, \ldots$$

3.91 The probability of an event as rare or rarer than the one observed can be calculated by using the hypergeometric distribution.

$$P(\text{one or fewer black members}) = \frac{\binom{8}{1}\binom{12}{5}}{\binom{20}{6}} + \frac{\binom{8}{0}\binom{12}{6}}{\binom{20}{6}} = \frac{8(792)}{38{,}760} + \frac{924}{38{,}760} = .187$$

This is not a very unlikely event, since it has probability close to $\frac{1}{5}$. It could very well have happened by chance. There is little reason to doubt the randomness of the selection.

3.96 There are N animals in the total population. After taking a sample of k animals, marking and releasing them, there are $N-k$ unmarked animals. We then choose a second sample of size 3 from the N animals. There exist $\binom{N}{3}$ ways of choosing this second sample and there are $\binom{N-k}{2}\binom{k}{1}$ ways of finding exactly one of the originally marked animals. For $k=4$, the probability of finding just one marked animal is

$$\frac{\binom{N-4}{2}\binom{4}{1}}{\binom{N}{3}} = \frac{\frac{(N-4)(N-5)(4)}{2}}{\frac{N(N-1)(N-2)}{6}} = \frac{12(N-4)(N-5)}{N(N-1)(N-2)}$$

Calculating this probability for various values of N, we find that its value is maximized for $N=11$ or $N=12$.

N	4	5	6	7	8	9	10	11	12	13
Prob.	.000	.000	.200	.343	.429	.476	.500	.509	.509	.503

3.98 Let Y be the number of customers arriving. Then Y follows a Poisson distribution with $\lambda = 7$. We perform the calculations exactly, however one could just as easily used table 3 appendix III.

a. $P(Y \le 3) = p(0) + p(1) + p(2) + p(3) = \frac{7^0 e^{-7}}{0!} + \frac{7^1 e^{-7}}{1!} + \frac{7^2 e^{-7}}{2!} + \frac{7^3 e^{-7}}{3!} = .0818.$

b. $P(Y \ge 2) = 1 - P(Y \le 1) = 1 - \frac{7^0 e^{-7}}{0!} - \frac{7^1 e^{-7}}{1!} = 1 - 8e^{-7} = .9927.$

c. $P(Y = 5) = \frac{7^5 e^{-7}}{5!} = .1277.$

3.102 The probabilities of 0, 1, 2, or 3 cars arriving at a particular entrance are shown in the table below.

y	Entrance I ($\lambda = 3$)	Entrance II ($\lambda = 4$)
0	.0497871	.0183156
1	.1493612	.0732626
2	.2240418	.1465251
3	.2240418	.1953668

Each is calculated by using a Poisson distribution with mean $\lambda = 3$ or $\lambda = 4$. Then

$$\begin{aligned} P(3 \text{ cars}) &= P(0 \text{ through I, 3 through II}) + P(1 \text{ through I, 2 through II}) \\ &\quad + \ldots + P(3 \text{ through I, 0 through II}) \\ &= .0497871(.1953668) + \ldots + (.2240418)(.0183156) = .0521 \end{aligned}$$

3.108 Let Y denote the number of deaths in 200 fires. Then Y has an approximate Poisson distribution with $\lambda = 3$. We use Table 3, Appendix III.

a. $P(Y > 8) = 1 - P(Y \le 8) = 1 - .996 = .004.$

b. Yes. If the region's rate is equal to the national average of 3 then a very rare event has occurred (see part **a**). We suspect that the region's rate is higher than 3 per 200 fires.