2.57 **a.** $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.1}{.3} = \frac{1}{3}$

b. $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{.1}{.5} = \frac{1}{5}$

c. $P(A|A \cup B) = \frac{P(A)}{P(A \cup B)} = \frac{.5}{.5+.3-.1} = \frac{5}{7}$

Notice that we are using the additive law of probability, given in Section 2.8.

d. $P(A|A \cap B) = \frac{P(A \cap B)}{P(A \cap B)} = 1$

e. $P(A \cap B|A \cup B) = \frac{P(A \cap B)}{P(A \cup B)} = \frac{.1}{.5+.3-.1} = \frac{1}{7}$

2.58 The necessary probabilities can be obtained directly from the table. To determine whether or not A and M are independent, look at

$$P(A) = .6 \quad \text{and} \quad P(A|M) = \frac{P(A \cap M)}{P(M)} = \frac{.24}{.4} = .6.$$

Hence A and M are independent. For $\overline{A}$ and F, look at

$$P(\overline{A}) = .4 \quad \text{and} \quad P(\overline{A}|F) = \frac{P(\overline{A} \cap F)}{P(F)} = \frac{.24}{.60} = .4.$$

Hence $\overline{A}$ and F are independent.

2.60 Define the following events:

U: job is unsatisfactory   and A: plumber A does the job

It is given that $P(A) = .4$, $P(U) = .1$, $P(A|U) = .5$.

a. The probability of interest is

$$P(U|A) = \frac{P(AU)}{P(A)} = \frac{P(A|U)P(U)}{P(A)} = \frac{.5(.1)}{.4} = .125.$$

b. $P(\overline{U}|A) = 1 - P(U|A) = 1 - .125 = .875.$

2.68 **a.** The three tests are independent. Thus, the probability in question is $(.05)^3 = .000125$.

b. The probability of at least one mistake equals 1 minus the probability of no mistakes. The probability of no mistakes is $(.95)^3$. Thus, the probability of at least one mistake is $1 - (.95)^3 = 1 - .857 = .143$.

2.73 **a.** $P(\text{current flows}) = 1 - P(\text{current is not flowing})$

$= 1 - P(\text{all three relays are open}) = 1 - (.1)^3 = .999$

b. Let A be the event that current flows and B be the event that the relay 1 closed properly

$P(B|A) = \frac{P(BA)}{P(A)} = \frac{P(B)}{P(A)}$; since $B \subset A$

$= \frac{.9}{.999} = .9009$

2.79 a. We assume that the Connecticut and Pennsylvania lotteries are independent. Thus,

P(666 in Connecticut$|$666 in Pennsylvania) $= P$(666 in Connecticut)

$$= \frac{1}{10^3} = .001$$

b. P(666 in Connecticut $\cap$ 666 in Pennsylvania)

$= P$(666 in Connecticut)P(666 in Pennsylvania)

$= (0.001)(1/8) = .000125$

2.80 $P(AB) = 1 - P(\overline{AB})$

$$= 1 - P(\overline{A} \cup \overline{B})$$ DeMorgan's Law

Aside: $P(\overline{A} \cup \overline{B}) \leq P(\overline{A}) + P(\overline{B})$ from addition law of probability

therefore, $P(AB) \geq 1 - P(\overline{A}) - P(\overline{B})$

2.81 P(landing safely on both jumps) $\geq 1 - .05 - .05 \geq .90$

2.86 Define the following events:

I: item comes from line I II: item comes from line II D: item is defective

Using the approach given in the solution to Exercise 2.57, write

$P(\overline{D}) = P[\overline{D} \cap (I \cup II)] = P(\overline{D} \cap I) + P(\overline{D} \cap II) = P(\overline{D}|I)P(I) + P(\overline{D}|II)P(II)$

$= .92(.4) + .90(.6) = .908.$

2.87 Define the following events:

A: buyer sees magazine ad

B: buyer sees corresponding ad on television

C: buyer purchases the product

The following probabilities are known:

$P(A) = .02$ $P(B) = .20$ $P(A \cap B) = .01.$

Now $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .02 + .20 - .01 = .21.$ Further,

$P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B) = 1 - .21 = .79$

where the event $\overline{A} \cap \overline{B}$ is the event that the buyer does not see the ad either on television or in a magazine. Finally, it is given that $P(C|A \cup B) = \frac{1}{3}$ and $P(C|\overline{A} \cap \overline{B}) = \frac{1}{10}$. It is necessary to find $P(C)$.

$P(C) = P$(buyer purchases the product)

$= P$(buyer sees ad and buys) $+ P$(buyer doesn't see ad and buys)

$= P[C \cap (A \cup B)] + P[C \cap (\overline{A} \cap \overline{B})]$

$= P(C|A \cup B)P(A \cup B) + P(C|\overline{A} \cap \overline{B})P(\overline{A} \cap \overline{B})$

$= \left(\frac{1}{3}\right)(.21) + \left(\frac{1}{10}\right)(.79) = .07 + .079 = .149.$

COMMENT:

Look at a population of 3000 potential buyers. Then the number of people :

Having seen the ad on magazine $= 3000 \times \frac{1}{50} = 60$

Having seen the ad on TV $= 3000 \times \frac{1}{5} = 600$

Having seen the ad on both $= 3000 \times \frac{1}{100} = 30$

Not seeing the ad $= 3000 - (60 + 600 + 30) = 2370$.

The number of people :

having seen the ad and buying the product $= (600 + 60 - 30) \times \frac{1}{3}$

$= 210$

not seeing the ad and buying the product $= 2370 \times \frac{1}{10} = 237$.

Hence the probability of a (random) customer buying the product is $\frac{237 + 210}{3000} = 0.149$.

2.90 Let $A\overline{A}$ = {positive reading for truthful person, negative reading for liar}.

Then the sample space is
$$S = \{AA, A\overline{A}, \overline{A}A, \overline{A}\,\overline{A}\}.$$
$$P(AA) = .10 \times .95 = .095$$
$$P(A\overline{A}) = .10 \times .05 = .005$$
$$P(\overline{A}A) = .9 \times .95 = .855$$
$$P(\overline{A}\,\overline{A}) = .9 \times .05 = .045$$

a. $P(AA) = .095$
b. $P(\overline{A}A) = .855$
c. $P(A\overline{A}) = .005$
d. $1 - P(\overline{A}\,\overline{A}) = .955.$

You've got to believe that both suspects said "I didn't do it."

2.96 Let D denote defective and G denote good.

a. The event of interest is the union of the following three mutually exclusive events:
$$DGGD \qquad GDGD \qquad GGDD$$
Note that the last defective must be found on the fourth test, but the other may be found on test 1, 2, or 3.

Consider the first event. A defective must be drawn on the first test. This occurs with probability $\frac{2}{6}$. A nondefective must be drawn on each of the next two tests; the probabilities are $\frac{4}{5}$ and $\frac{3}{4}$, respectively. A defective must be drawn on the fourth test. This happens with probability $\frac{1}{3}$. The probability of this intersection is
$$\left(\tfrac{2}{6}\right)\left(\tfrac{4}{5}\right)\left(\tfrac{3}{4}\right)\left(\tfrac{1}{3}\right).$$
The probabilities associated with the other two events are identical to that of the first. Hence, applying the additive law of probability, we obtain the desired probability:
$$3 \times \tfrac{2}{6} \times \tfrac{4}{5} \times \tfrac{3}{4} \times \tfrac{1}{3} = 3 \times \tfrac{4\cdot3\cdot2\cdot1}{6\cdot5\cdot4\cdot3} = \tfrac{1}{5}.$$

b. We must locate the second defective refrigerator on the second, third, or fourth test. Call this event A. Consider the following events:

A_1: the second defective is found on the second test
A_2: the second defective is found on the third test
A_3: the second defective is found on the fourth test

Then $A = A_1 \cup A_2 \cup A_3$ is the union of three mutually exclusive events and
$$P(A) = P(A_1) + P(A_2) + P(A_3)$$
$P(A_3) = \frac{1}{5}$ was found in part a. The event $A_1 = DD$ is the only way to obtain the second defective on the second test, and
$$P(A_1) = P(DD) = \tfrac{2}{6} \times \tfrac{1}{5} = \tfrac{1}{15}$$
A_2 is the union of the two mutually exclusive events, DGD or GDD, which occur with equal probabilities
$$P(DGD) = P(GDD) = \tfrac{4}{6} \times \tfrac{2}{5} \times \tfrac{1}{4} = \tfrac{1}{15} \qquad \text{so} \qquad P(A_2) = \tfrac{2}{15}$$
Thus,
$$P(A) = P(A_1) + P(A_2) + P(A_3) = \tfrac{1}{15} + \tfrac{2}{15} + \tfrac{1}{5} = \tfrac{2}{5}$$

c. One of the two defectives has been found in the first two tests. Thus there are three nondefectives and one defective remaining. The other defective must be found on the third or fourth test. Call this event B, and express it as the union of two mutually exclusive events defined below:

B_1: the second defective is found on the third test
B_2: the second defective is found on the fourth test

Now the probability of event B_1 is $P(B_1) = \frac{1}{4}$. Also, the probability of event B_2, which is the intersection of a nondefective on the third draw and a defective on the fourth, is $P(B_2) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$. The event of interest is
$$P(B) = P(B_1 \cup B_2) = P(B_1) + P(B_2) = \tfrac{1}{4} + \tfrac{1}{4} = \tfrac{1}{2}.$$

2.97

a. $\frac{1}{n}$

b. $\left(\frac{n-1}{n}\right) \times \left(\frac{1}{n-1}\right) = \frac{1}{n}$ second try

    $\left(\frac{n-1}{n}\right) \times \left(\frac{n-2}{n-1}\right) \times \left(\frac{1}{n-2}\right) = \frac{1}{n}$ third try

c. $P[\text{gain access}] = P(\text{first try}) + P(\text{second try}) + P(\text{third try})$

                  $= \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = \frac{3}{7}$

2.99 Define these events:

D: person has the disease   and H: test says person has the disease

Then $P(H|D) = .9$; $P(\overline{H}|\overline{D}) = .9$; $P(D) = .01$; $P(\overline{D}) = .99$. Using Bayes's Rule,

$$P(D|H) = \frac{P(H|D)P(D)}{P(H|D)P(D)+P(H|\overline{D})P(\overline{D})} = \frac{(.9)(.01)}{(.9)(.01)+(.1)(.99)} = \frac{.009}{.108} = \frac{1}{12}.$$

$$P(D|\overline{H}) = \frac{P(\overline{H}|D) \cdot P(D)}{P(\overline{H}|D) \cdot P(D) + P(\overline{H}|\overline{D}) \cdot P(\overline{D})} = \frac{0.1 \times 0.01}{0.1 \times 0.01 + 0.9 \times 0.99}$$

2.104 Define these events:

C: contract lung cancer S: worked in a shipyard

Then $P(S|C) = .22$ and $P(S|\overline{C}) = .14$. Also, $P(C) = .0004$. Using Bayes's Rule,

$$P(C|S) = \frac{P(S|C)P(C)}{P(S|C)P(C)+P(S|\overline{C})P(\overline{C})} = \frac{(.22)(.0004)}{(.22)(.0004)+(.14)(.9996)} = \frac{.000088}{.140032}$$

$$= .0006$$

2.111 Define these events:

G: student guesses C: student correctly answers question

We know $P(G) = .2$, $P(C|\overline{G}) = 1$, and $P(C|G) = .25$. Thus,

$$P(\overline{G}|C) = \frac{P(C|\overline{G})P(\overline{G})}{P(C|\overline{G})P(\overline{G})+P(C|G)P(G)} = \frac{(1)(.8)}{(1)(.8)+(.25)(.2)} = \frac{.8}{.85} = .9412$$

2.114 Let A = woman's name selected from list 1 and B = woman's name selected from list 2.

Then

$$P(A) = \frac{5}{7}, P(\overline{B}|A) = \frac{6}{9}, P(\overline{B}|\overline{A}) = \frac{7}{9}.$$

Now

$$P(A|\overline{B}) = \frac{P(\overline{B}|A)P(A)}{P(\overline{B}|A)P(A)+P(\overline{B}|\overline{A})P(\overline{A})} = \frac{\left(\frac{6}{9}\right)\left(\frac{5}{7}\right)}{\left(\frac{6}{9}\right)\left(\frac{5}{7}\right)+\left(\frac{7}{9}\right)\left(\frac{2}{7}\right)} = \frac{30}{44}.$$