AM 165 Assignment II Solutions

2.18

a. Let N_1 and N_2 be the empty cans and let W_1 and W_2 be the cans filled with water. Then the pair (N_1W_2) will be the simple event that the expert chooses cans N_1 and W_2. The simple events in S are listed below.

$E_1 = (N_1N_2)$ $\quad E_3 = (N_1W_2)$ $\quad E_5 = (N_2W_2)$
$E_2 = (N_1W_1)$ $\quad E_4 = (N_2W_1)$ $\quad E_6 = (W_1W_2)$

b. If the rod is worthless the expert is merely guessing so each simple event is equally likely. Hence

$P(E_i) = \frac{1}{6}$, $i = 1, 2, \ldots, 6$

and the probability that the expert picks the two cans containing water is

$P(E_6) = \frac{1}{6}$.

2.22

a. Let $w1$ denote the first wine, $w2$ the second, and $w3$ the third. Then one sample point would be an ordered triple indicating the rank of each wine. For example, $(w1, w2, w3)$ would indicate that $w1$ is superior to $w2$and $w3$ while $w2$ is superior to just $w3$.

b. The sample space is given by all of the possible ordered triples. That is,

$$(w1,\ w2,\ w3),\ (w1,\ w3,\ w2),\ (w2,\ w1,\ w3),$$
$$(w2,\ w3,\ w1),\ (w3,\ w1,\ w2),\ (w3,\ w2,\ w1)$$

c. Suppose $w1$ is superior to $w2$ and $w3$. Then the probability that the "expert" ranks $w1$ as first or second is

$P[(w1, w2, w3)$ or $(w1, w3, w2)$ or $(w2,\ w1,\ w3)$ or $(w3,\ w1,\ w2)] = 4/6 = 2/3$.

2.23

a. Two systems are selected from six, two of which are defective. Denote the six systems as $G_1, G_2, G_3, G_4, D_1, D_2$, according to whether they are defective or nondefective. Each sample point will represent a particular pair of systems chosen for testing. The sample space, consisting of 15 pairs, is shown below.

G_1G_2 $\quad G_1G_3$ $\quad G_1G_4$ $\quad G_1D_1$ $\quad G_1D_2$ $\quad G_2G_3$ $\quad G_2G_4$ $\quad G_2D_1$
G_2D_2 $\quad G_3G_4$ $\quad G_3D_1$ $\quad G_3D_2$ $\quad G_4D_1$ $\quad G_4D_2$ $\quad D_1D_2$

Note that the two systems are drawn simultaneously and that order is unimportant in identifying a sample point. Hence the pairs G_1G_2 and G_2G_1 are not considered to represent two different sample points. Then

$P(\text{at least one defective}) = \frac{9}{15} = \frac{3}{5}$ and $P(\text{both defective}) = P(D_1D_2) = \frac{1}{15}$.

b. If four of the six systems are defective, the 15 sample points are

G_1G_2 $\quad G_1D_1$ $\quad G_1D_2$ $\quad G_1D_3$ $\quad G_1D_4$ $\quad G_2D_1$ $\quad G_2D_2$ $\quad G_2D_3$
G_2D_4 $\quad D_1D_2$ $\quad D_1D_3$ $\quad D_1D_4$ $\quad D_2D_3$ $\quad D_2D_4$ $\quad D_3D_4$

Then

$P(\text{at least one defective}) = \frac{14}{15}$ and $P(\text{both defective}) = \frac{6}{15} = \frac{2}{5}$.

2.27

The mn rule is used. The flight from New York to California can be chosen in any one of 6 ways, the flight from California to Hawaii in any one of 7 ways. Thus, the total number of flights will be $(6)(7) = 42$.

2.29

a. There are $6! = 720$ possible itineraries.

b. If Denver is the first city visited then there are 5 ways for San Francisco to follow and 4! ways to arrange the other 4 cities. If Denver is the second then there are 4 ways for San Francisco to follow and 4! ways to arrange the other cities and so on. Thus, there are 4!(15) ways for Denver to precede San Francisco. Hence

$$P(\text{Denver before San Francisco}) = 360/720 = 0.5.$$

2.31

a. Use the mn rule. Since the first die can result in one of 6 possible outcomes, and the second can result in one of 6 outcomes, a total of $(6)(6) = 36$ pairs can be formed, each containing one element from the first group and one element from the second.

b. Define the event A, "observe a sum of 7 on the two dice." This event will occur if any one of the following simple events occurs:

$(1, 6),\quad (2, 5),\quad (3, 4),\quad (4, 3),\quad (5, 2),\quad (6, 1)$

Then $P(A) = P(\text{observe a sum of } 7) = \frac{6}{36} = \frac{1}{6}$.

2.35 Use mn rule. The total ways are

$$\binom{9}{3} \times \binom{6}{5} \times \binom{1}{1} = 504 \text{ ways.}$$

2.37. Use mn rule. The coefficient should be

$$\binom{17}{2} \times \binom{15}{5} \times \binom{10}{10} = 408,408$$

Or we can use Theorem 2.3:

$$\binom{17}{2\ 5\ 10} = 408,408$$

2.38. The coefficient of x^5y^3 should be

$$\binom{8}{5\ 3} = \binom{8}{5} = 56$$

Similarly, the coefficient of x^3y^5 is

$$\binom{8}{3\ 5} = \binom{8}{3} = 56$$

2.39 a. The total ~~#~~ number of double majors is

$$\binom{130}{2} = 8385$$

b. There are 26 choice for each letter. Use mn rule, the total number for two-letter code is

$26 \times 26 = 676$

The total number of three-letter codes is

$$26 \times 26 \times 26 = 17576$$

So total number is

$$17576 + 676 = 18252$$

i.e. there are 18252 total major codes available.

c. Total required is

$$130 + \binom{130}{2} = 130 + 8385 = 8515$$

d. Yes.

2.40. Two numbers 4 and 6 are possible for each of the three digits. An extension of the mn rule gives

$$2 \times 2 \times 2 = 8$$

potential winning three-digit numbers.

2.44 The total number of ways to choose 4 students from 8 is $\binom{8}{4} = \frac{8!}{4!4!} = 70$. Since the choice is random, each of the 70 sample points is equally likely, and it remains only to determine how many sample points result in exactly 2 of the 3 undergraduates and 2 of the 5 graduates. Using the mn rule, this number is $(C_2^3)(C_2^5) = 3(10) = 30$ and the desired probability is $\frac{30}{70} = \frac{3}{7}$.

2.50 The total possibilities of tossing a die for six times are

$$6 \times 6 \times 6 \times 6 \times 6 \times 6 = 6^6$$

The possibilities of all six numbers being recorded are

$$6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6!$$

so. the probability is

$$\frac{6!}{6^6} = \frac{5}{324}$$