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Applied Mathematics 165

Course Outline

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I. Introduction

A. Inference and statistics

1. **Examples**
2. **Estimation**
3. **Hypothesis Testing**

B. Sampling problem

C. Role of probability theory

II. Probability

A. Probability and statistics

1. **Intuitive Notions: relative frequencies**
2. **Dual roles of probability and statistics**

B. The probability space

1. **Sample space, S**
2. **Events**
3. **Probability**

C. Calculating probabilities: combinatorial approach

1. **Brief introduction to combinatorial rules**
2. **Examples**

D. Calculating probabilities: event-composition approach

1. **Decomposing an event**
2. **Conditional probability & independence**
3. **Total probability formula**
4. **Bayes' rule**

E. Random variables

1. **Introduction**
2. **Discrete random variables**

a) Distribution of a random variable

b) Some commonly occurring discrete distributions

   (1) Binomial distribution

   (2) Geometric distribution

   (3) Hypergeometric distribution

   (4) Poisson distribution

   (a) Definition and description of situations it models

   (b) Approximation of the binomial distribution

c) Expected value (Expectation)

   (1) Definition

(2) Important expectations characteristic of a distribution
(a) Mean
(b) Variance and standard deviation
(3) Simplifying formulas
(a) E[c]
(b) E[cg(X)]
(c) $\Sigma_i g_i(X)$]
(4) Tchebysheff's inequality
(a) Implications for statistics: convergence of relative frequencies
(b) The empirical rule

3. Continuous random variables

a) Cumulative distribution function
b) Density functions
c) Some commonly occurring continuous distributions
(1) Uniform distribution
(2) Normal (Gaussian) distribution
(a) Definition
(b) Calculating probabilities for normal distributions; standardization
(3) Gamma family of distributions
(a) Definition
(b) Exponential distribution; lack-of-memory property
(c) Chi-squared distribution
d) Expected value (Expectations)
(1) Definition
(2) Important expectations characteristic of a distribution
(a) Mean
(b) Variance and standard deviation
(3) Linearity of expected value
(4) Tchebysheff revisited

4. Relations between different random variables

a) Introduction
b) Independence
(1) Definition
(2) Examples
c) Expected value of functions of several random variables
d) Linear dependence, covariance and correlation
(1) Statistical motivation: simple linear regression
(2) Definition and interpretations of covariance and correlation
(3) Variance of sums of random variables

III. Statistics

A. Introduction to statistics

1. Goals of statistics
2. Random sampling
3. Examples

B. Estimation

1. Parameter estimation

a) Formulation
b) Some commonly used estimators

(1) Estimating means and their differences
(2) Estimating probabilities
(3) Estimating variances
(4) Errors in estimation: how close is an estimate to the target parameter

c) Systematic approaches to estimation

(1) Method of moments
(2) Method of maximum likelihood
(3) "Plug-in" estimators

2. Confidence-interval estimation

a) Goal and formulation
b) Rule-of-thumb: the "empirical rule"
c) Large-sample confidence intervals

(1) Small sample vs. large sample statistics
(2) Law of Large Numbers
(3) The Central Limit Theorem
(a) Statement
(b) Relation to the empirical rule
(c) Sums of independent random variables
(4) Constructing a large-sample confidence interval
(5) Choice of sample size

d) Commonly used small-sample confidence intervals for samples from a Normal population

(1) Preliminaries: the chi-square and *t* distributions
(2) Confidence intervals for the mean
(3) Confidence intervals for a difference of means
(4) Confidence interval for the variance

C. Hypothesis Testing

1. Introduction

a) Formulation
b) Type I error and choice of the critical region
c) Type II error and choice of sample size

2. Some commonly used tests

a) Large sample tests and the central limit theorem
b) Small sample tests for samples from a Normal population

(1) Tests concerning the mean
(2) Tests concerning a difference of means
(3) Tests concerning the variance
(4) (Tests concerning a ratio of variances)

3. Comments on systematic procedures for constructing a test: the Neyman-Pearson Lemma

4. Examples from "Nonparametric" statistics

a) The chi-square goodness-of-fit test
b) The sign test for a paired experiment