ROLLS AND HEXAGONS

BIFURCATION THEORY WITH SYMMETRY

Experiments (Rayleigh-Bernard convection):

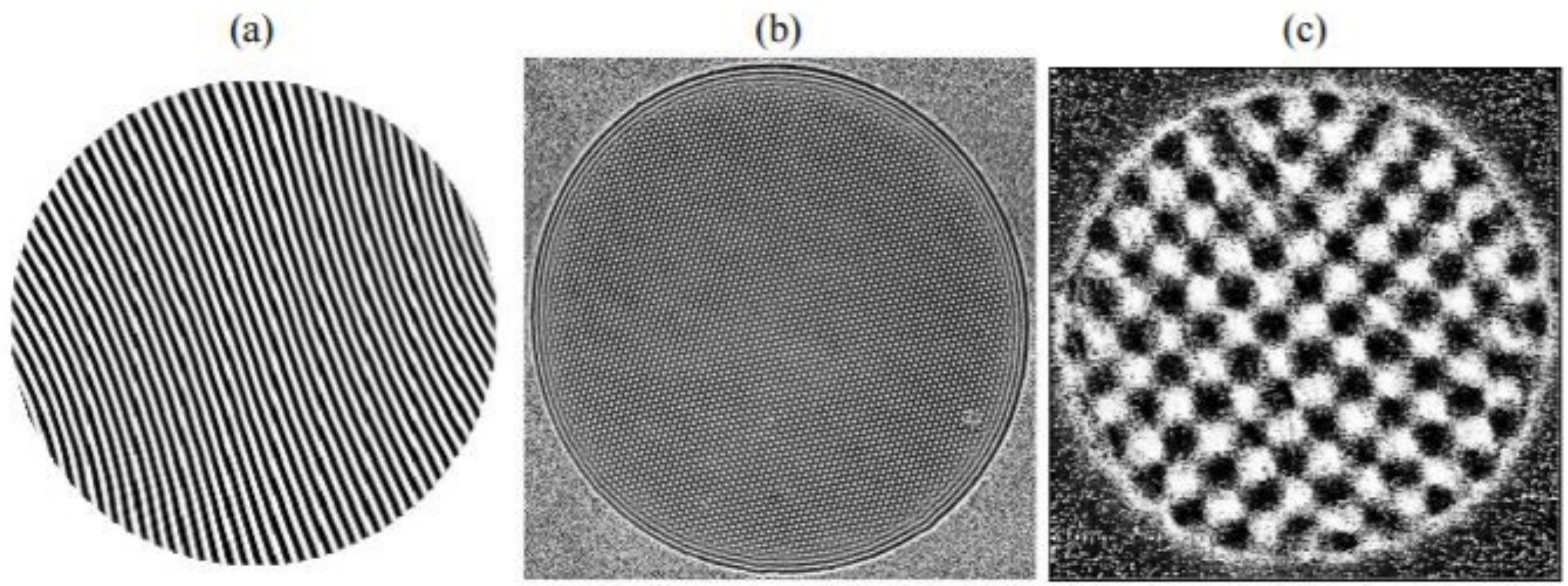


Fig. 4.1 Convection patterns. (a) Rolls; (b) Hexagons; (c) Squares.

http://pauli.uni-muenster.de/tp/fileadmin/lehre/NumMethoden/SoSe10/Skript/SH.pdf

Amplitude equation reduces to the

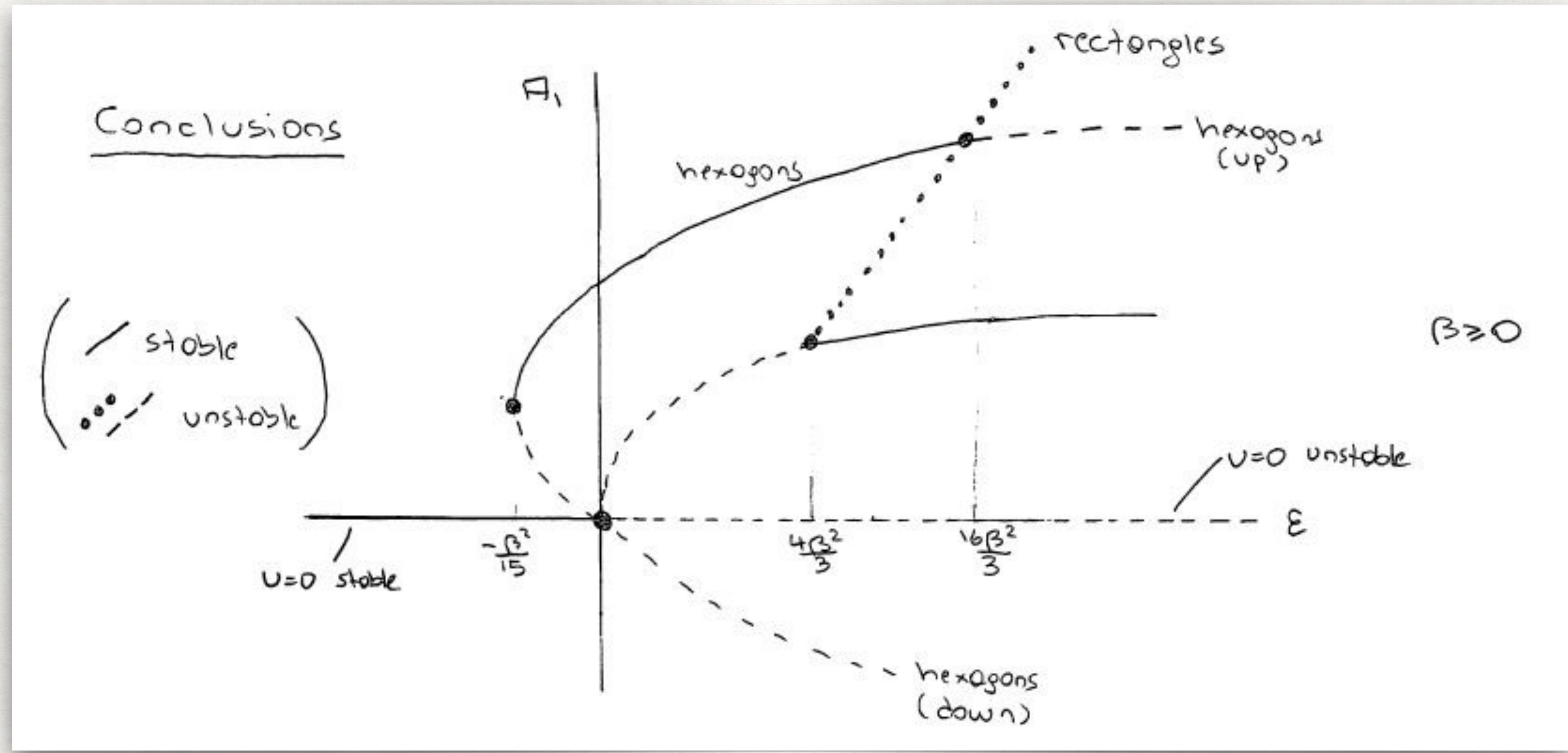
Generalized Swift-Hohenberg equation:

$$u_t = -(1+\Delta)^2 u + \epsilon u + \beta u^2 - u^3$$

where

$$x \in \mathbb{R}^2,\ u \in \mathbb{R}\,.$$

Move up to the bifurcation diagram:



Lattice patterns:

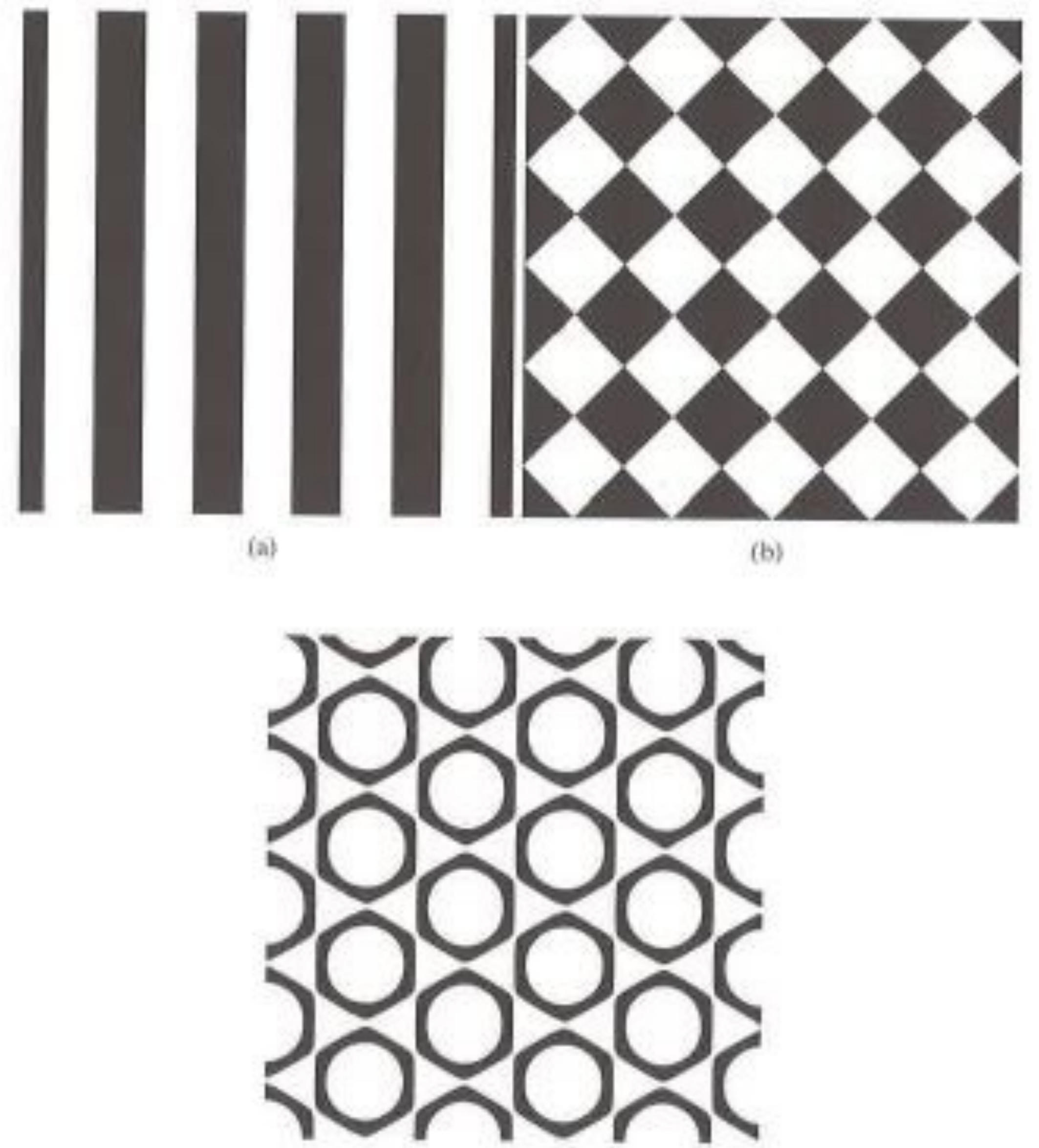


Fig. 5.1. Examples of some of the common lattice patterns: (a) stripes or rolls, (b) squares and (c) hexagons. These are filled contour plots of $u = \sum_j (e^{i\mathbf{k}_j \cdot \mathbf{x}} + e^{-i\mathbf{k}_j \cdot \mathbf{x}})$ for (a) $\mathbf{k}_1 = (1, 0)$, (b) $\mathbf{k}_1 = (1, 0)$, $\mathbf{k}_2 = (0, 1)$, and (c) $\mathbf{k}_1 = (1, 0)$, $\mathbf{k}_2 = (-1/2, \sqrt{3}/2)$, $\mathbf{k}_3 = (-1/2, -\sqrt{3}/2)$, and so are purely linear superpositions of stripe patterns at various angles to each other. For the stripes and squares, $u > 0$ regions are shown in white and $u < 0$ in black. For the hexagons the central white region is $u > 0$, the black region is $-1.95 < u \leq 0$, and the outer white region is $u \leq -1.95$. In experiments there are usually some higher harmonics present, leading the patterns to look a little different even when the symmetries are the same.

Hoyle 2006, Pattern Formation: An introduction to methods

Solutions on the hexagonal lattice:

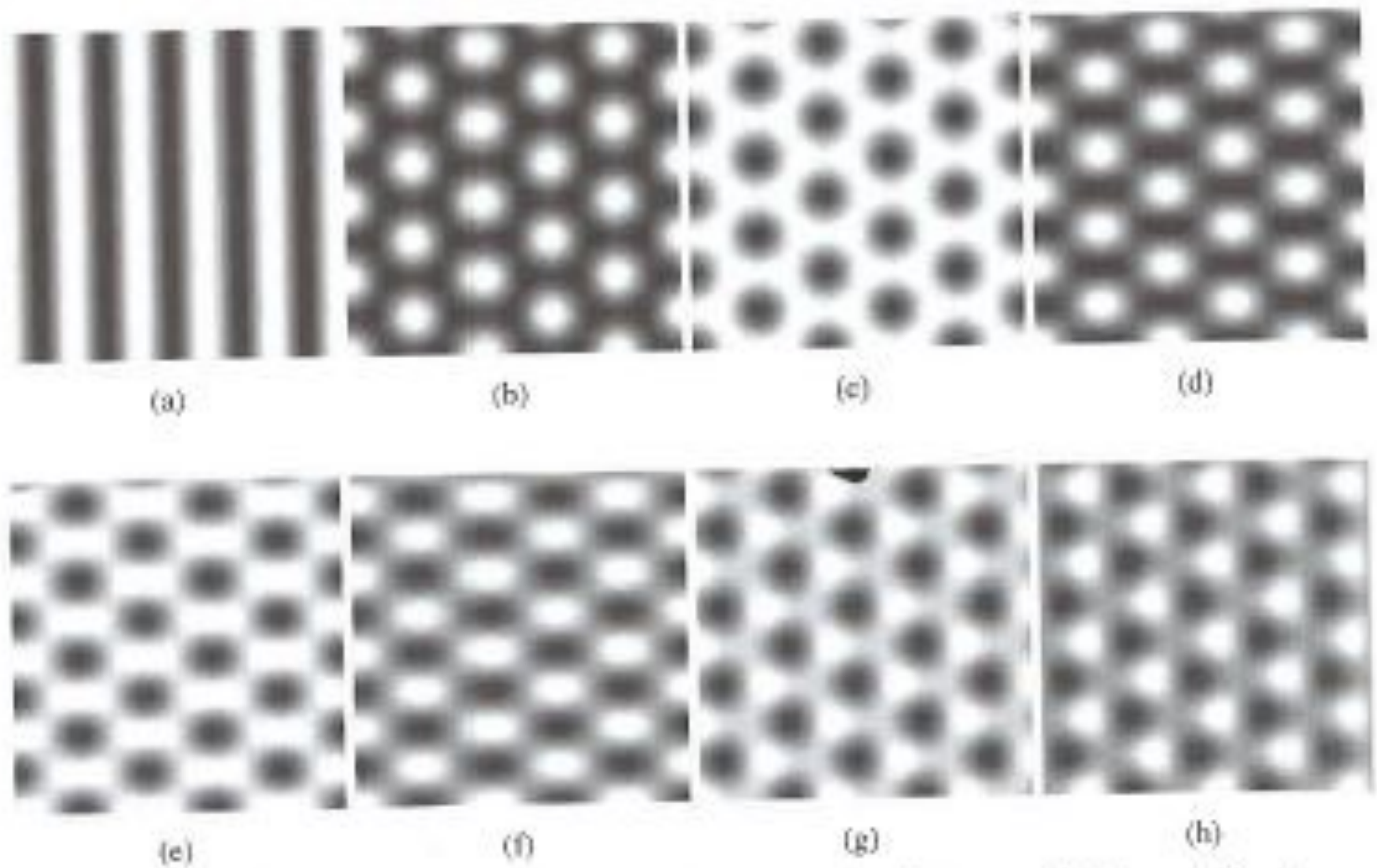


Fig. 5.7. Greyscale plots of some solutions on the hexagonal lattice: (a) rolls, (b) up hexagons, (c) down hexagons, (d) up rectangles, (e) down rectangles, (f) patchwork quilt, (g) triangles and (h) regular triangles.

Hoyle 2006, Pattern Formation: An introduction to methods