Derivation of macroscopic equations for individual cell-based models: A formal approach

Bodnar and Velasquez, 2005

November 10, 2015

Notations and a few assumptions

Notations

- N = number of cells
- V: potential
- $X_k(t)$: position of the center of cell k at time t
- $\xi_k(t)$: uncorrelated white noises
- d: average distance between cells
- R: range of interactions of potentials

Notations and a few assumptions

Assumptions

- $\rho(x,t) = \frac{\text{the number of cells in } [x, x+\Delta x] \text{ at time } t}{N}$
- $d \sim 1/N$
- $d \ll \Delta x \ll l$

I.e., consider $X_N(t) = \frac{1}{N}\sum_{k=1}^{N} \delta_{X_k^N(t)}$ and consider the weak limit $X_N(t) \to \rho(x,t)dx$ as $N \to \infty$.

- $R \approx d$: short range interactions
- $R \gg d$: long range interactions

Main model equation

Model equation:

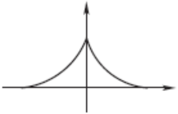
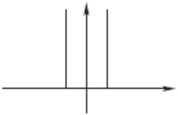
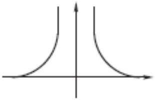
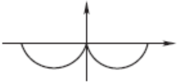
$$\frac{\mathrm{d}X_k^N(t)}{\mathrm{d}t} = -\frac{1}{N}\sum_{\substack{i=1\\ i\neq k}}^{N} \nabla V_N(X_k^N(t) - X_i^N(t)) + \sigma\xi_k(t), \quad k = 1,\dots,N$$

Scaling of potential:

$$V_N(x) = N^{\beta} V_1(N^{\beta x}), \qquad 0 < \beta \leq \ .$$

Summary of results

Table I. Summary of the macroscopic limits.

$V(x)$	σ	Range	Limit equation	Derivation—in section
	0	$d \ll R$	$\rho_t = D_1(\rho^2)_{xx}$	3.1
	0	$d \simeq R$	$\rho_t = \frac{1}{2}(\rho D(\rho))_{xx}$	3.1
	$\neq 0$	$s \gg R$	$\rho_t = D_1(\rho^2)_{xx} + \sigma^2 \rho_{xx}$	3.2
	0	$d \simeq R$	$\rho_t = 0$	3.2.2
	$\neq 0$	$d \simeq R$	$\rho_t = \left(\sigma^2 \frac{\rho_x}{(1-c\rho)^2}\right)_x$	3.2.2
	$\neq 0$	$d \gg R$	$\rho_t = D_1(\rho^2)_{xx} + \sigma^2 \left(\frac{\rho_x}{(1-c\rho)^2}\right)_x$	3.2.3
	0	$d \gg R$	$\rho_t = \left(\rho \int V(x-y)\rho(y)\,\mathrm{d}y\right)_x$	4