Graph Applications

GirlsGetMath 2016

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Outline

- **Graph coloring**: useful in scheduling
- **Clustering**: useful in understanding social networks
- **Epidemics/rumor spread** on networks: useful in understanding how diseases as well as news spread in communities

1. Graph Coloring

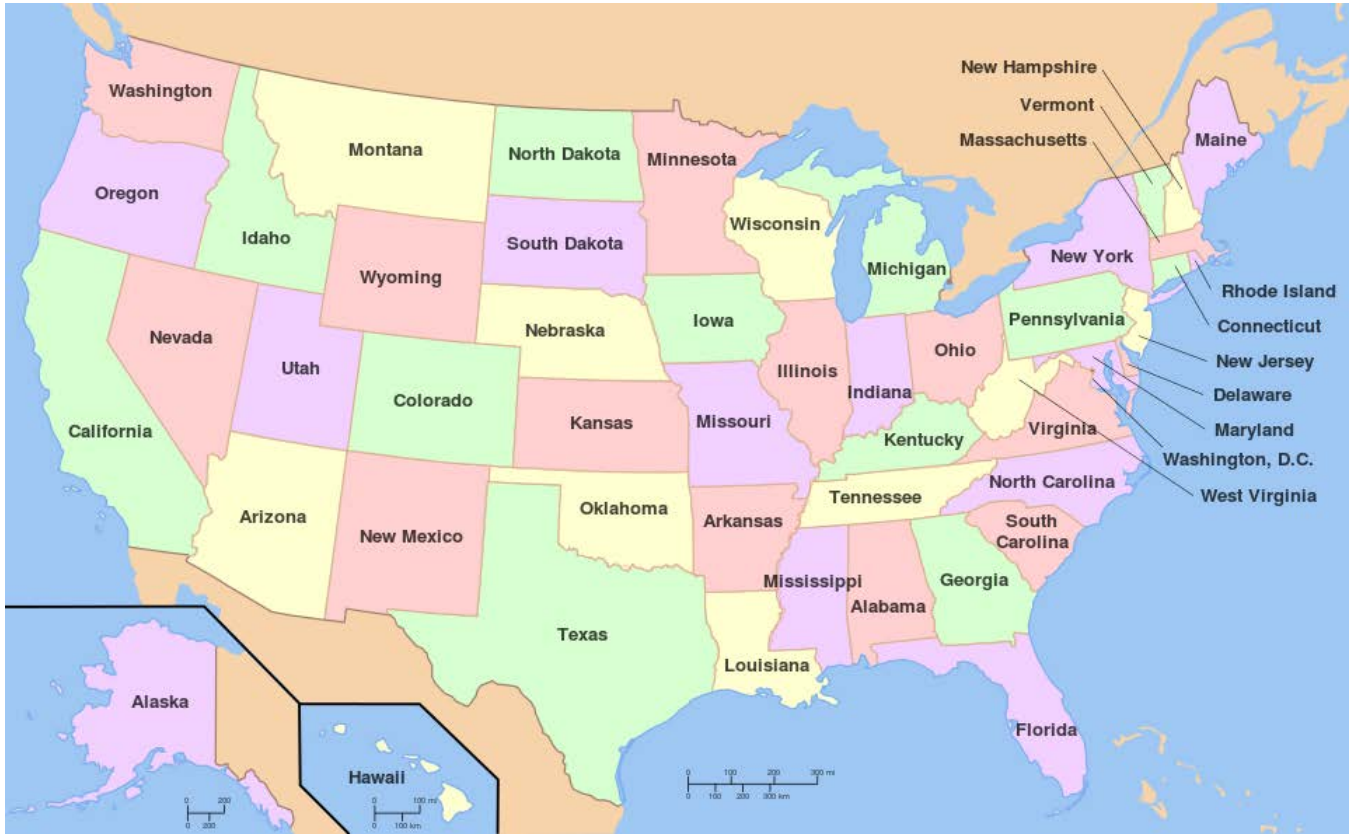
- Consider the following question (no graphs yet!):

Given a map of a country divided into different territories, what is the fewest number of colors needed to color each region so that no adjacent territories have the same color?

- Example: United States map

- How many colors are needed?

- Example: United States map



- 4 colors!

Four Color (Map) Theorem

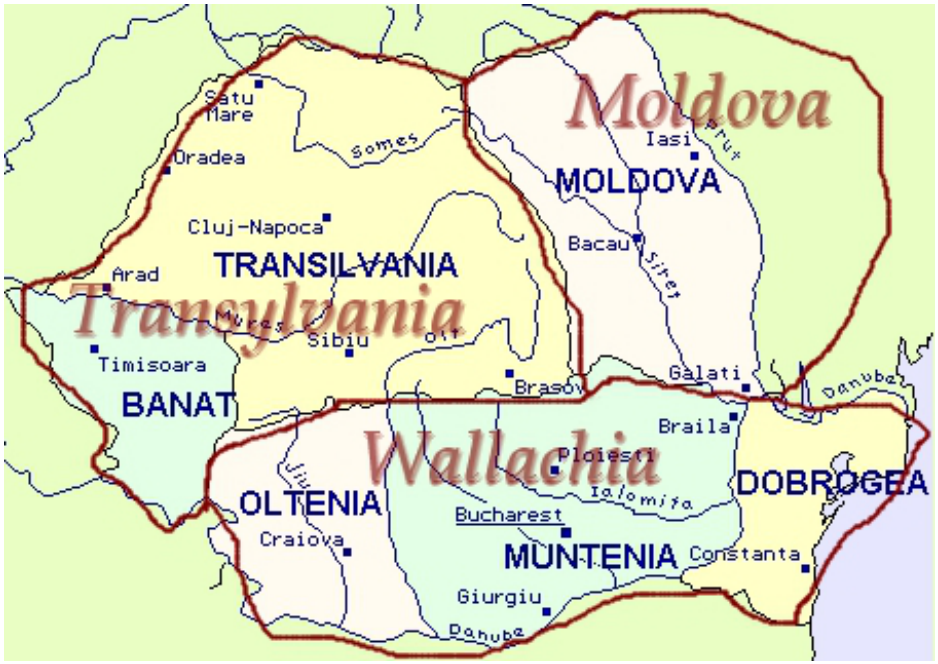
Given any separation of a plane into contiguous regions, producing a figure called a map, no more than **four** colors are required to **color** the regions of the map so that no two adjacent regions have the same color**.**

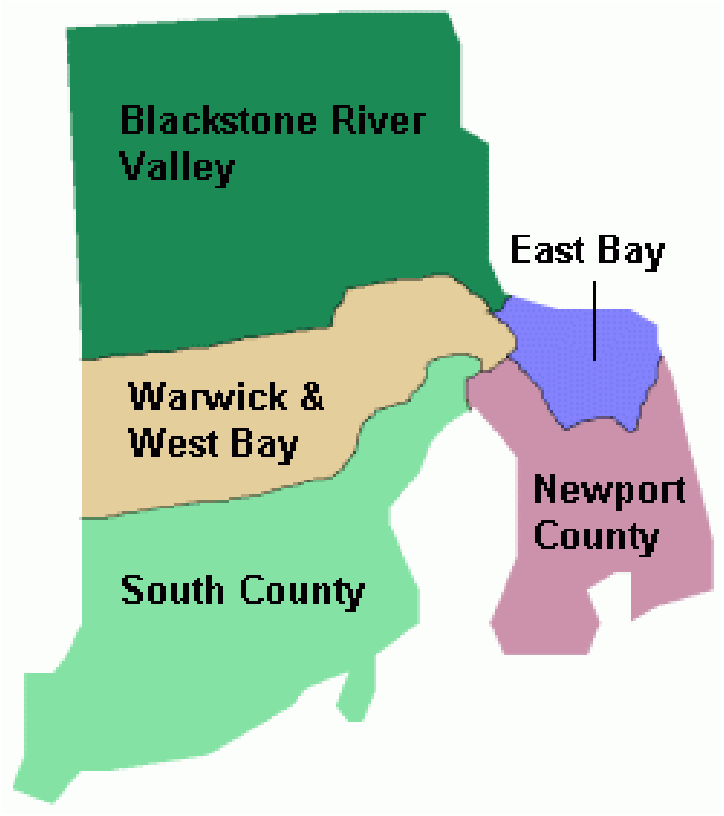
- **Proof:** computer-assisted, using **graph theory**
- The min number of colors is called **chromatic number**

Graph theory approach

- How could we phrase the map coloring problem using graphs? What would the vertices correspond to? How about the edges?

- Take 1 -2 minutes to discuss this with your neighbor, and try it with the worksheet maps.



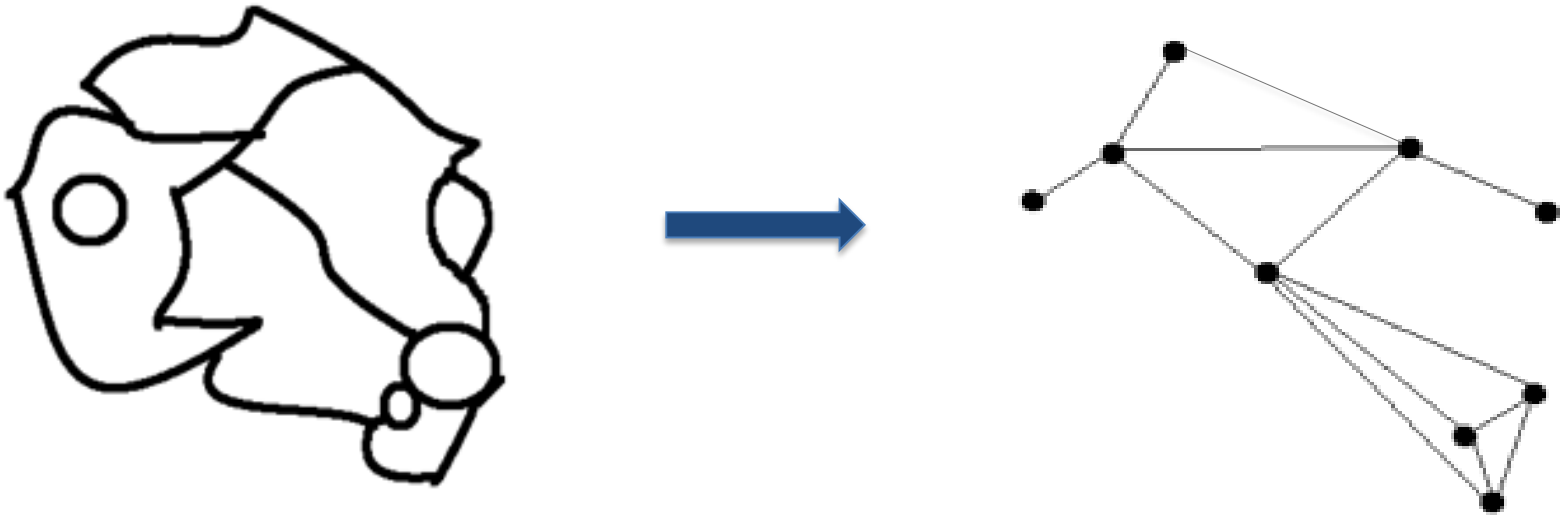


Romania: 3 regions

RI: 5 regions

Worksheet 1.1

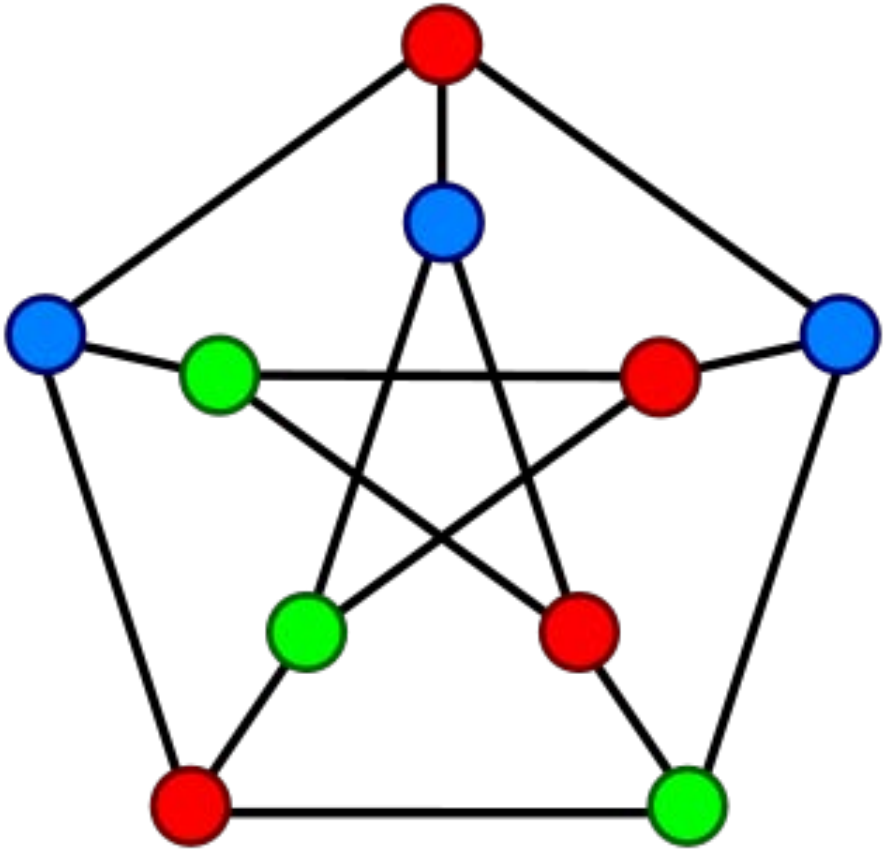
- Consider a fictional country with regions, and label each region with a vertex. Then connect vertices if the two regions share a border.



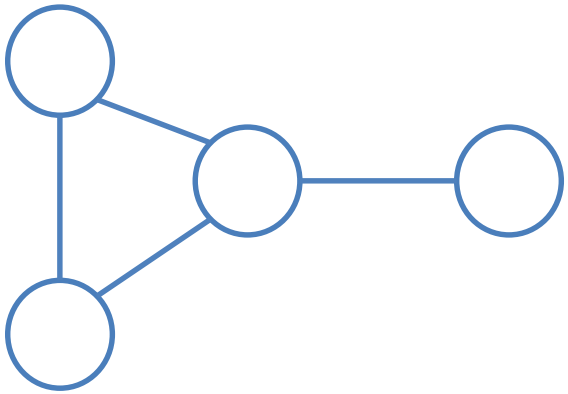
- Consider a fictional country with regions, and label each region with a vertex. Then connect vertices if the two regions share a border.

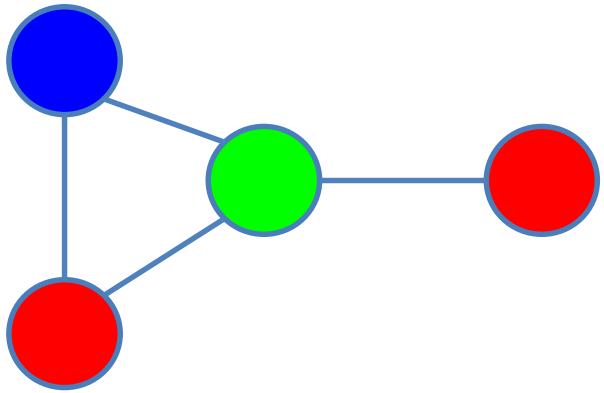
- Chromatic number = 4

- But there are graphs that require fewer colors:



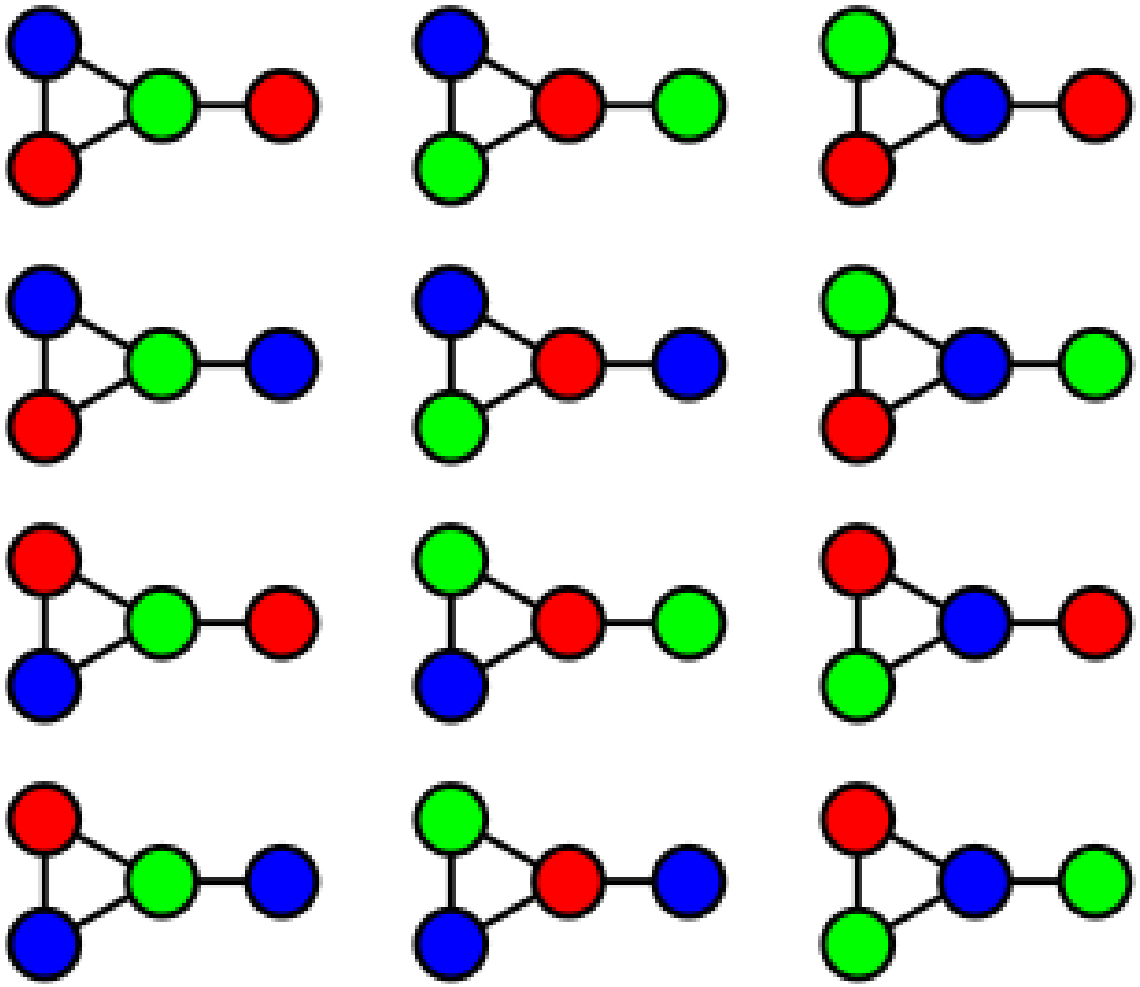
Chromatic number = 3





- On the provided sheet, find all possible 3-colorings of this graph!

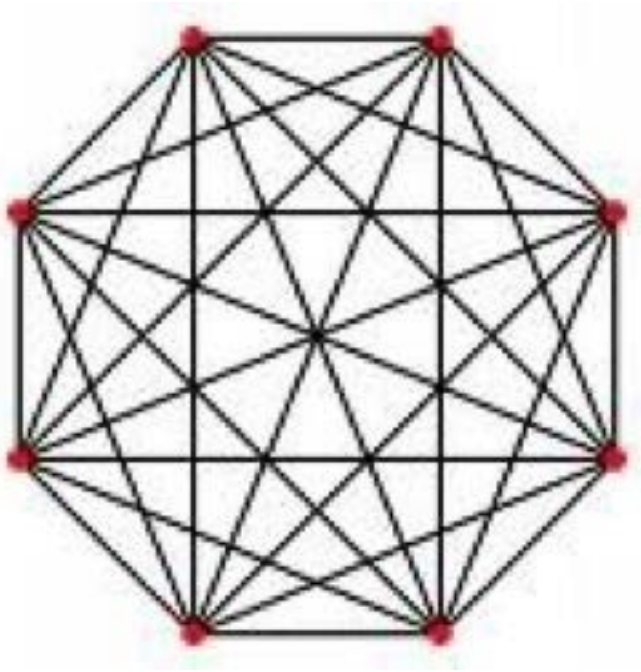
Worksheet 1.2



Worksheet 1.2

- There are also graphs that require more colors.
- Recall the meaning of a complete graph.

Start with complete graphs with 4,5,6, … vertices. How many colors do you need?

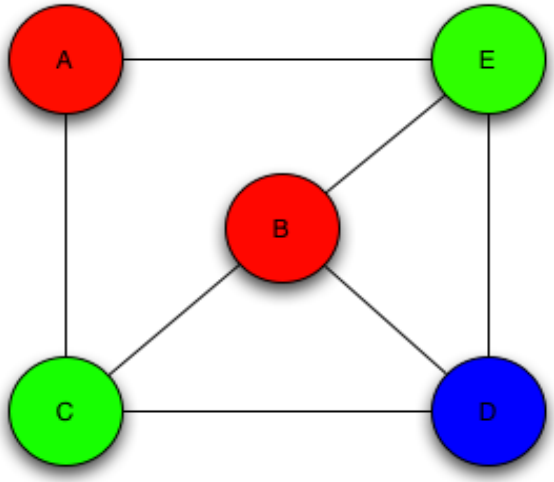


How about a complete graph with n vertices?

Worksheet 1.3

Applications of Coloring Problem

- Scheduling: exams need to be assigned to time slots at a school: same student may take different subjects



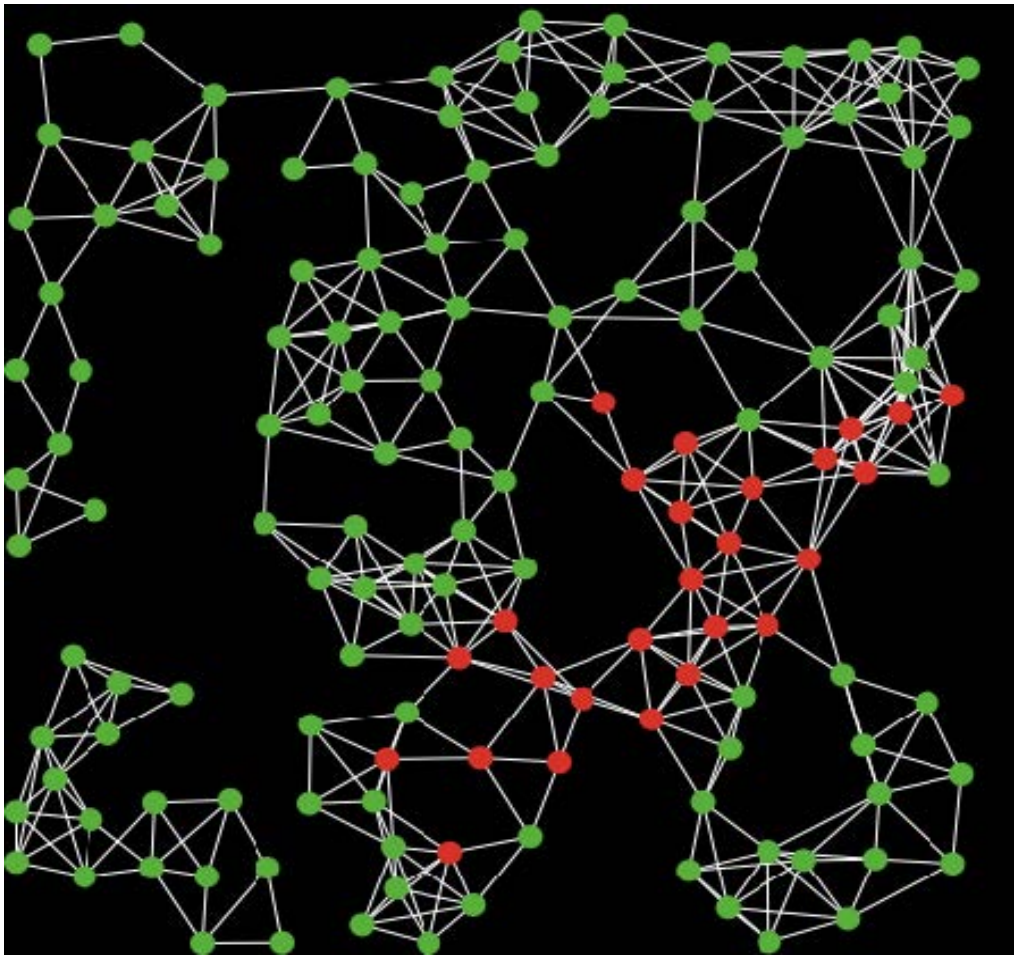
- What could the vertices, edges, and colors correspond to? Discuss with your neighbor.

- Vertices: subjects, edges: students who take both subjects, colors: time slots for exams

- What is the meaning of the chromatic number?

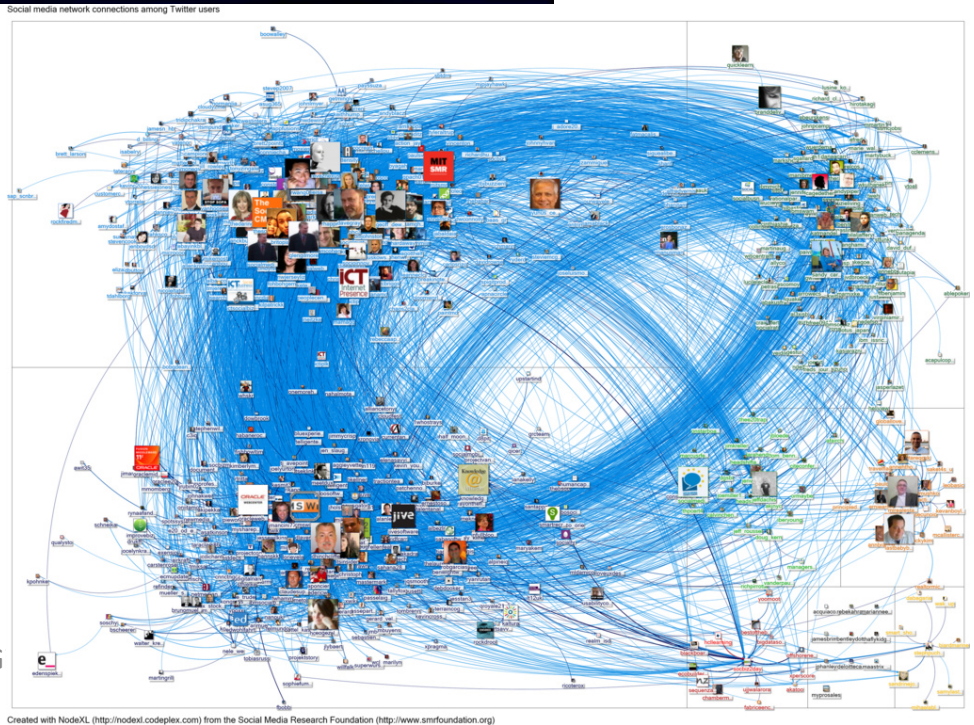
- Graph coloring results for the chromatic number can be used to find the number of time slots necessary for the exams

2. Rumor/Disease Spreading



Facebook

Twitter



G

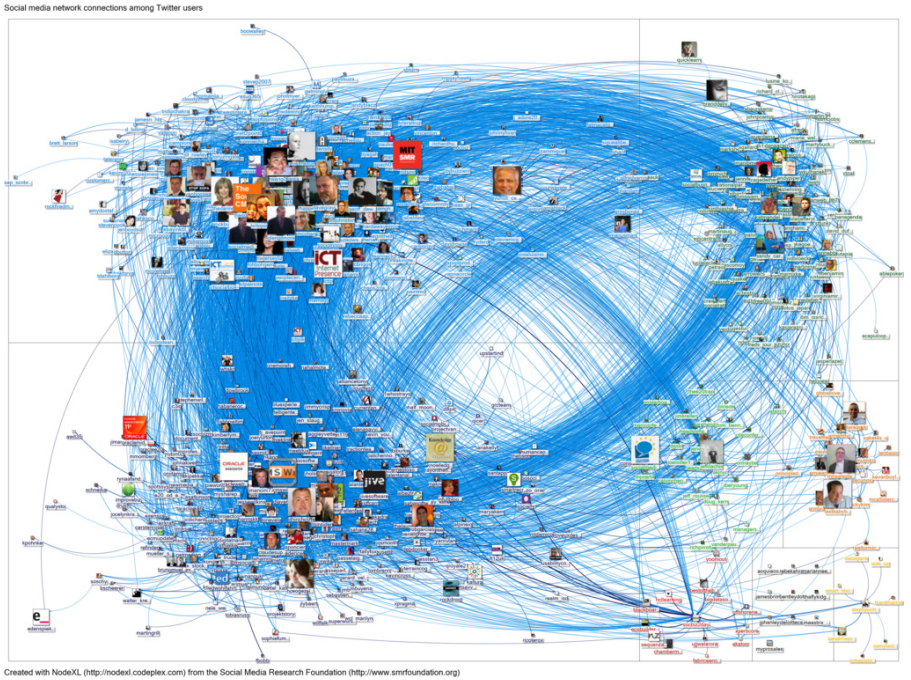
Graph/ network	Epidemic spread	Rumor/news spread
Vertices		
Edges		

- Difference: what is being spread (disease vs. news)

Worksheet 2.1

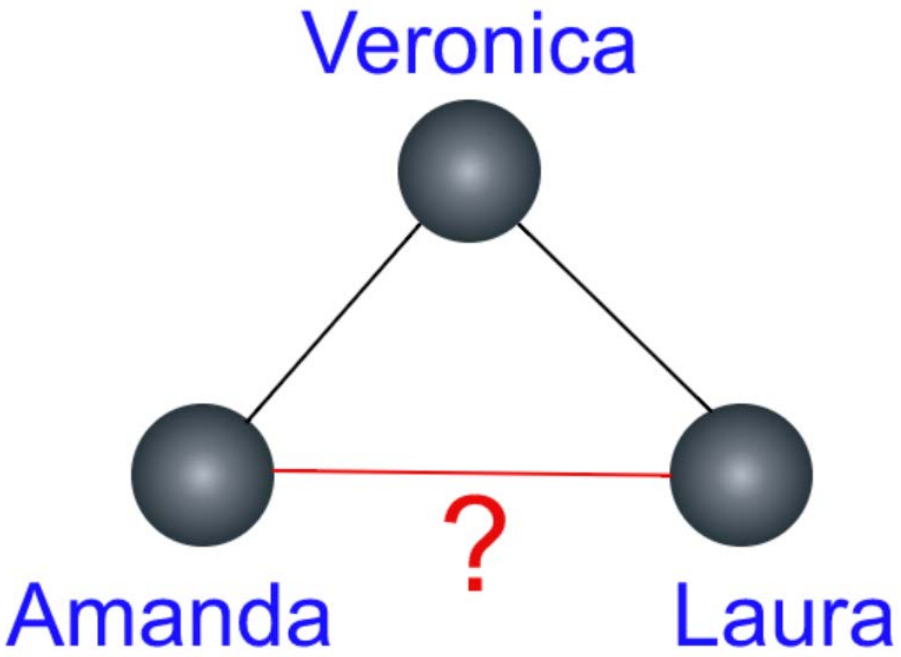
Network types

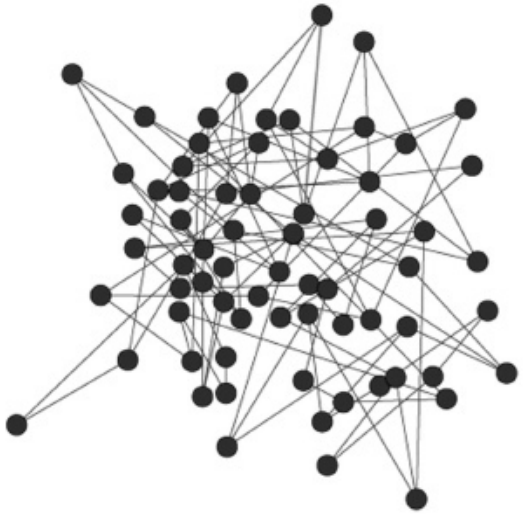
- Facebook and Twitter are **social network** examples
- Which properties do you expect these networks to have: directed, undirected, sparse, highly connected?
- Discuss with your neighbor

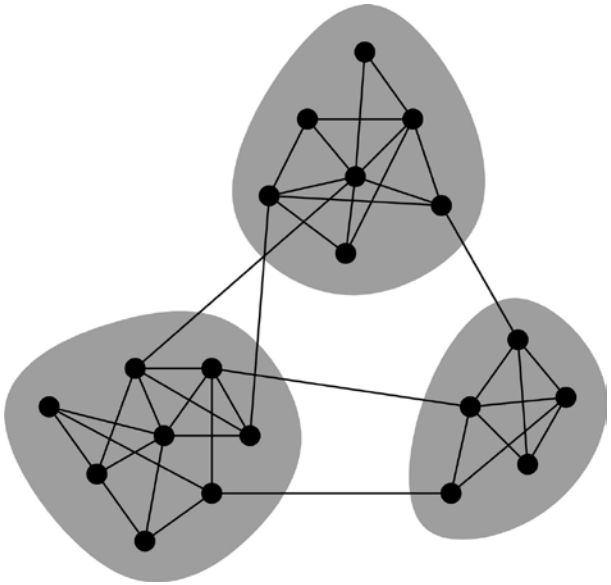


Clustering

- Interesting property of networks: clustering
- Clustering measures how **clique**-ish a network is
- I.e., if I have two friends, how likely is it that they are also friends?







Not clique-ish (random)

Very clique-ish

Do you think
Facebook is clique-ish?

- We can calculate the local clustering coefficient of a **node**:

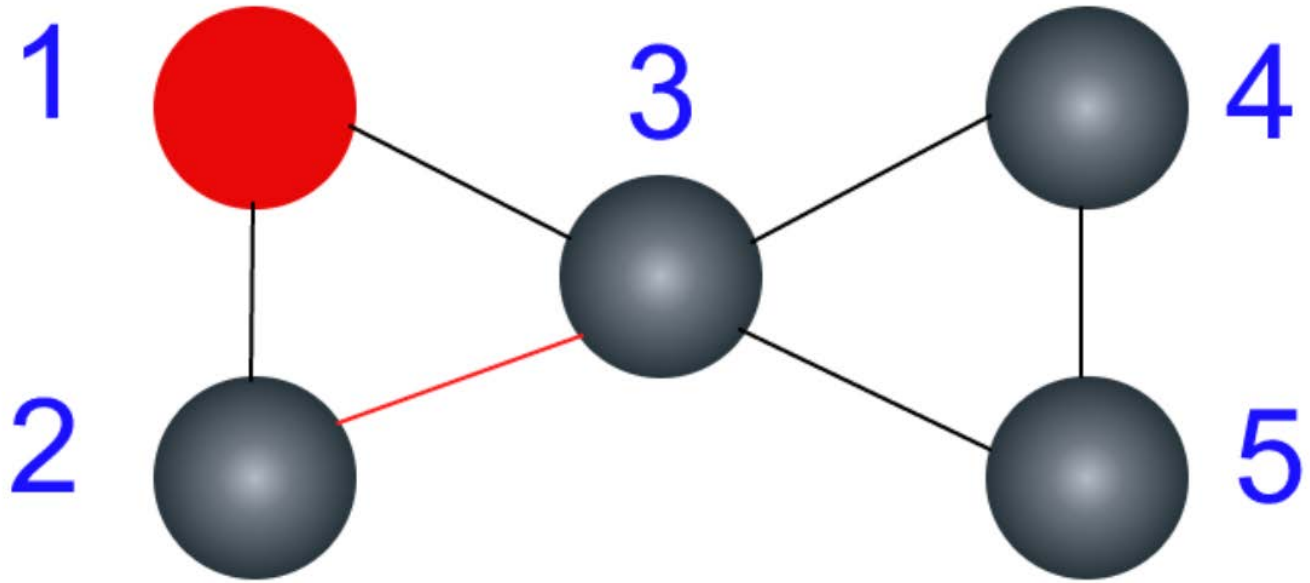
$$cl_i = \frac{number\ of\ pairs\ of\ neighbors\ connected\ by\ edges}{number\ of\ pairs\ of\ neighbors}$$

$$= \frac{number\ of\ pairs\ of\ neighbors\ connected\ by\ edges}{d_i * (d_i - 1)/2}$$

Where d_i is the degree of node i.

Network clustering coefficient $\overline{cl}$: average of the individual clustering coefficients of each node.

$$cl_i = \frac{number\ of\ pairs\ of\ neighbors\ connected\ by\ edges}{d_i * (d_i - 1)/2}$$



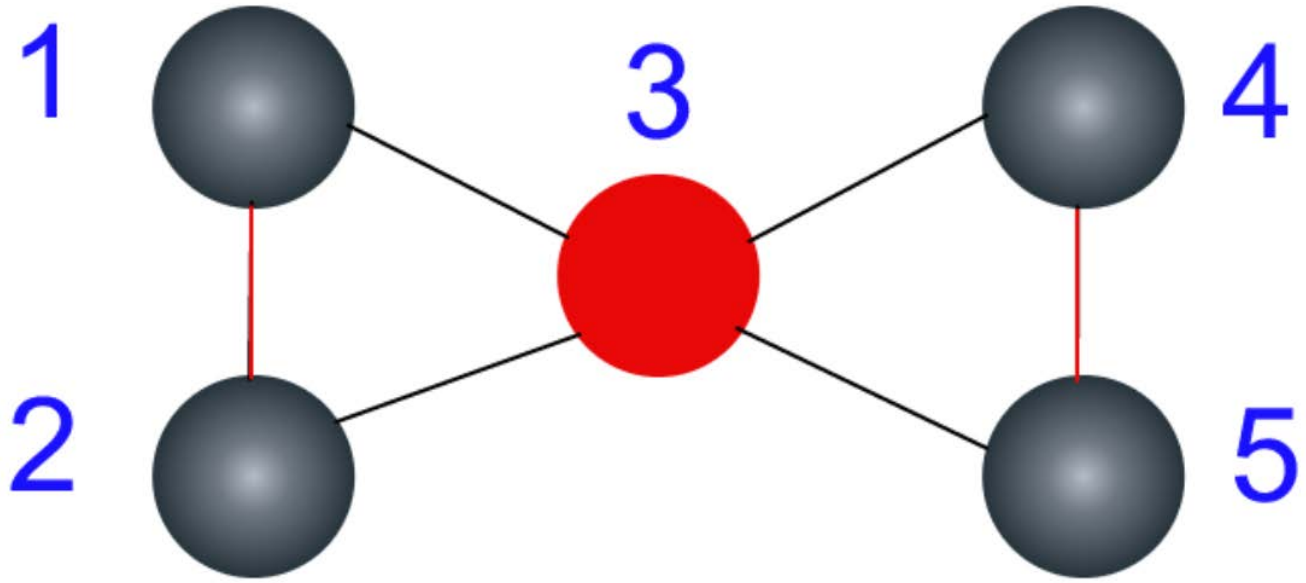
$$cl_1 = \frac{1}{2 * (2-1)/2} = \frac{1}{2/2} = 1$$

$$cl_2 = \frac{1}{2 * (2-1)/2} = \frac{1}{2/2} = 1$$

How about cl_2?

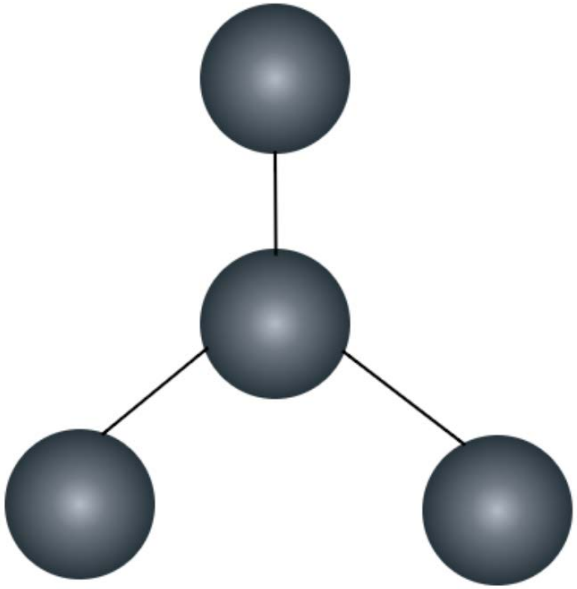
Worksheet 2.2

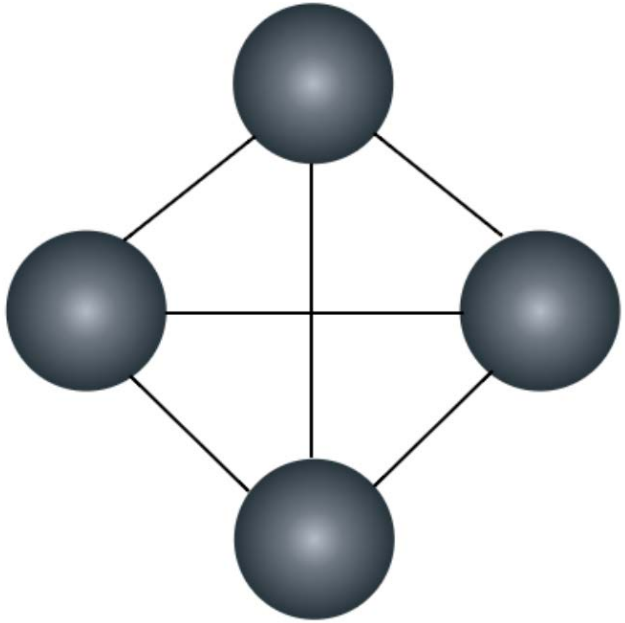
$$cl_i = \frac{number\ of\ pairs\ of\ neighbors\ connected\ by\ edges}{d_i * (d_i - 1)/2}$$



$$cl_3 = \frac{2}{4 * (4-1)/2} = \frac{2}{12/2} = \frac{2}{6} = \frac{1}{3}$$

Average network clustering: $\frac{1+1+1+1+\frac{1}{3}}{5} = 0.867$





$\overline{cl} = 0$

$\overline{cl} = 1$

Not clique-ish

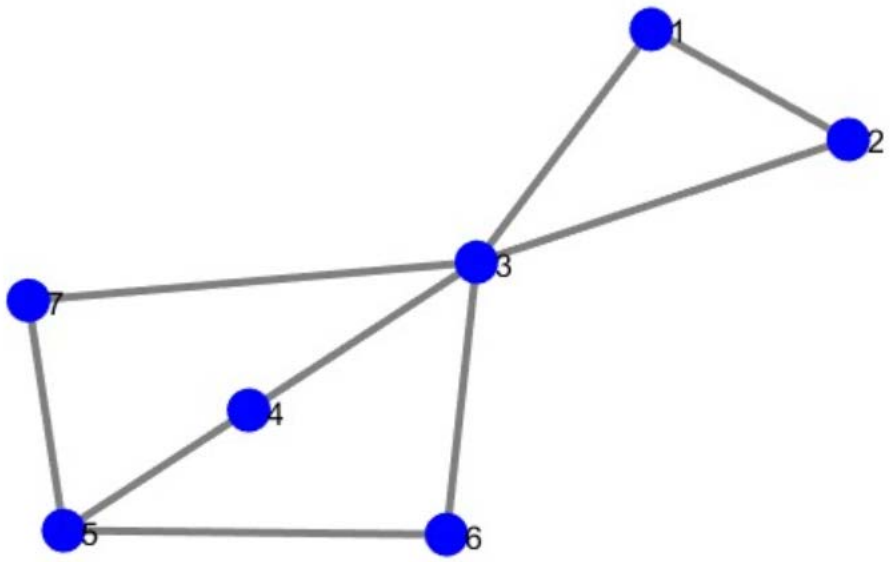
Very clique-ish

Clustering coefficient: 0.15-0.6
(depends on the specific community)

Example: epidemic spread

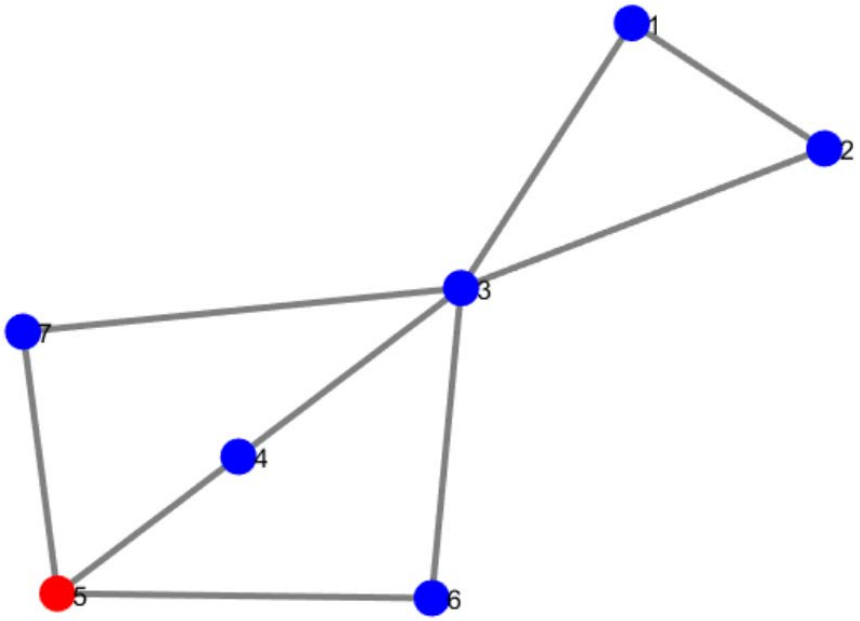
Adjacency matrix:

0	1	1	0	0	0	0
1	0	1	0	0	0	0
1	1	0	1	0	1	1
0	0	1	0	1	0	0
0	0	0	1	0	1	1
0	0	1	0	1	0	0
0	0	1	0	1	0	0



Worksheet 2.3 (Challenge)

- Epidemic starts through an infected individual/seed (red nodes)



- How fast the disease spreads depends on the probability that an infected individual infects a susceptible one.

- Question of interest: Does the speed of transmission depend on the infected seed?

- In graph theory, this question is referred to as **centrality**: how important is a node in spreading the disease/news in the network.

- It turns out there are many ways to calculate centrality; which one is most appropriate for different networks is still a subject of research.

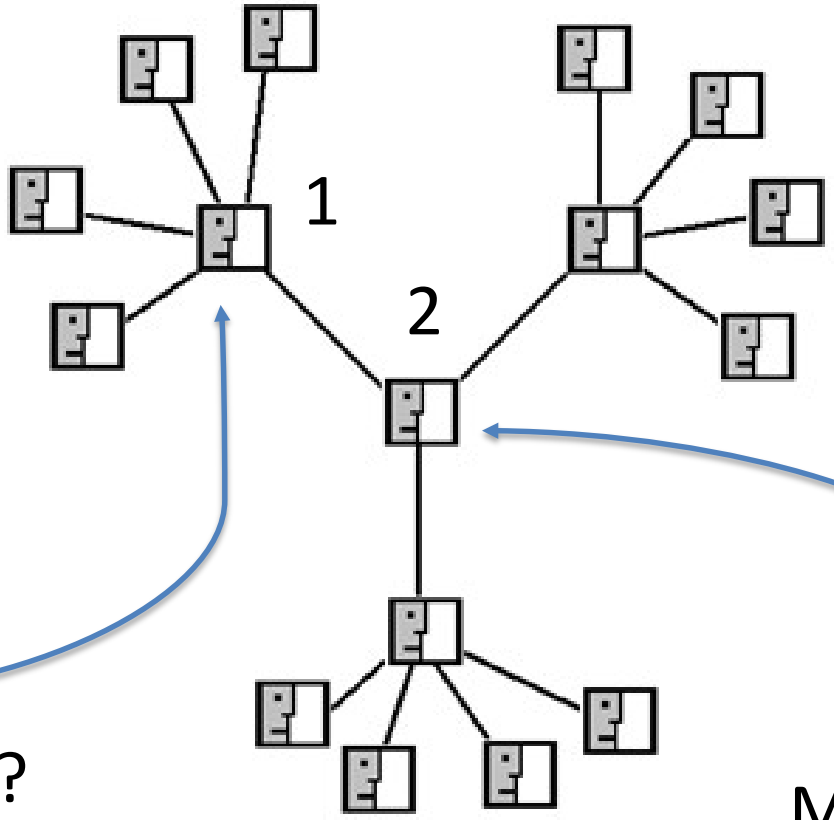
Degree Centrality

- How many connections/followers/friends do I have?

- For node i:

$$c_i^d = \frac{degree\ of\ node\ i}{sum\ of\ all\ node\ degrees}$$

- Note that this centrality measure only depends on local neighbor information.



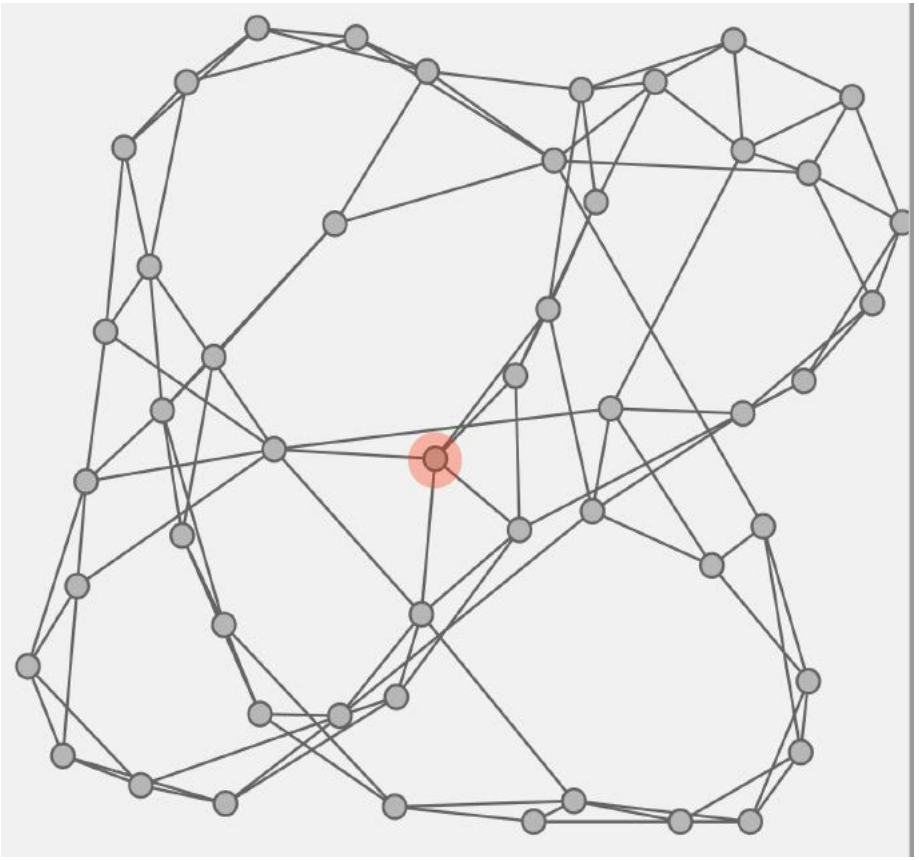
Most central?
Local degree information

Most central?
Global information

- Suppose we know which node has the highest centrality.

- How is this helpful in the case of an outbreak?

- How about in the case of embarrassing news about you?



Bibliography

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