APMA 0160 (A. Yew) Spring 2011

Homework #7

Deadline: April 17 (Sunday) 4:00am EST

Problem 1

Write a Matlab program that implements the **composite 3-node Gauss–Legendre quadrature rule**, computing approximations to $\int_a^b f(x)\,dx$ by repeatedly bisecting the integration interval and using a 3-node Gauss–Legendre rule on each half. As the stopping criterion, require that successive approximations differ by no more than a given tolerance; each time the interval is halved, require that the approximations on the two subintervals lie within half the tolerance for the original interval. The program should also count the total number of function evaluations done.

(i) Use your code to compute $\displaystyle\int_0^{48} \sqrt{1+\cos^2 x}\,dx$ with a tolerance of 10^{-5}. Save the computed value of the integral and the number of function evaluations (in that order) as a row vector in `A11.dat`.

(ii) Use your code to compute $\displaystyle\int_{10}^{19} \sin(t^2)\,dt$ with a tolerance of 10^{-5}. Save 10^5 times the computed value of the integral along with the number of function evaluations as a row vector in `A12.dat`.

(iii) Use your code to compute the cumulative integral $\displaystyle\int_0^x e^{-t^2}\,dt$ for $x = 0,\ 0.02,\ 0.04, \ldots, 0.98,\ 1$ with a tolerance of 10^{-5}. Save the values of the cumulative integral as a column vector in `A13.dat`.

Problem 2

Repeat Problem 1 for the **composite 5-node Gauss–Lobatto quadrature rule**. Save your part (i) result in `A21.dat`, your part (ii) result in `A22.dat`, and your part (iii) result in `A23.dat`.

Problem 3

Download `velocity.txt` from the Homework section of the course webpage (the same as `velocity.dat` that was used as an example in a previous class). This file contains data on the velocity of an object (in meters per second) measured at a sequence of times (seconds). The first column lists the times and the second column contains the corresponding velocities.

Find the displacement as a function of time; this involves integrating the velocity data with a varying upper limit of integration, i.e. computing the cumulative integral $\displaystyle\int_0^T v(t)\,dt$ for increasing values of T. Do this in the following ways:

(a) Use Matlab's built-in cumulative trapezoidal rule on the data. Save the results as a column vector in `A31.dat`.

(b) Fit a cubic spline interpolant with not-a-knot boundary conditions through the data points, and then use a trapezoidal rule to evaluate the cumulative integral for $T = 0 : 0.1 : 30$. Save the results as a column vector in `A32.dat`.

(c) Find the least-squares fit to the data with the function

$$F(t) = a\cos(bt) + ct + d,$$

using $a = 3$, $b = \pi/4$, $c = 2/3$, $d = 32$ as initial guesses for the parameters. Save the coefficients a, b, c and d (in that order) as a row vector in `A33.dat`. Then compute $\int_0^T F(t)\,dt$ for $T = 0 : 0.1 : 30$ using Matlab's built-in Simpson's rule function with a tolerance of 10^{-4}, and save the results as a column vector in `A34.dat`.