

Homework #6

Deadline: April 7 (Thursday) 4:00am EST

Problem 1

Download `muddy_signal.txt` from the Homework section of the course webpage. This file contains data sampled regularly over a fixed time interval from a signal that has been contaminated with noise. The first column lists the t -values (the sampling times) and the second column contains the corresponding y -values.

- Compute the power spectrum of the muddy signal, and save it in `A11.dat` as a column vector whose first entry is the power in Fourier mode zero, second entry is the power in the first mode, third entry is the power in the second mode, and so on.
- To filter out the noise, kill the Fourier modes with power less than 0.5. Extract the three dominant frequencies, in Hz, of the original (unperturbed) signal, and save them in ascending order as a column vector in `A12.dat`.
- Compute the y -values of the filtered signal, and save them as a column vector in `A13.dat`.

Problem 2

Write a Matlab program that implements **composite Boole's rule**, computing approximations to $\int_a^b f(x) dx$ by repeatedly bisecting the integration interval and using Boole's rule on each half. As the stopping criterion, require that successive approximations differ by no more than a given tolerance; each time the interval is halved, require that the approximations on the two subintervals lie within half the tolerance for the original interval. The program should also count the total number of function evaluations done.

- Use your code to compute $\int_0^{48} \sqrt{1 + \cos^2 x} dx$ with a tolerance of 10^{-5} . Save the computed value of the integral and the number of function evaluations (in that order) as a row vector in `A21.dat`.
- Use your code to compute $\int_{10}^{19} \sin(t^2) dt$ with a tolerance of 10^{-5} . Save 10^5 times the computed value of the integral along with the number of function evaluations as a row vector in `A22.dat`.
- For $p = 10, 10.5, 11, 11.5, \dots, 29.5, 30$, find the step size h your code uses to compute $\int_0^1 e^{-x^2} dx$ to within a tolerance of e^{-p} . By plotting $\ln(h)$ against p and calculating the gradient $\frac{\min \ln(h) - \max \ln(h)}{30 - 10}$, estimate the **order** of composite Boole's rule, i.e. the power q such that the error is proportional to h^q . Save your *unrounded* estimate of q in `A23.dat` with at least four decimal places.
- Now, based on the rounded value of your result from (iii), rewrite your code so that the computed value of the integral it outputs is *extrapolated* from two successive approximations; also, relax the tolerance requirement on the subintervals to increase computational efficiency. Use the modified code to compute $\int_{10}^{19} \sin(t^2) dt$ with a tolerance of 10^{-5} . Save 10^5 times the computed value of the integral along with the number of function evaluations as a row vector in `A24.dat`.

Problem 3

Repeat Problem 2 for the **composite 3-point open Newton-Cotes rule**. Save your part (i) result in `A31.dat`, your part (ii) result in `A32.dat`, your part (iii) result in `A33.dat` and your part (iv) result in `A34.dat`.