APMA 0160 (A. Yew) Spring 2011

Homework #5

Deadline: March 24 (Thursday) 4:00am EST
Scorelator will begin accepting submissions from March 17 (Thursday)

Problem 1

Download the file `temp.txt` from the Homework section of the course webpage (the same file of temperature data over 24 hours that was used in Homework #4).

(a) Find the *interpolating polynomial* for this data set. Evaluate it at the points `linspace(0,24,100)` and save these values as a column vector in `A11.dat`.

(b) Find the *piecewise linear* interpolant for the data set; this is a "curve" $S(x)$ composed of splines $S_1(x), S_2(x), \ldots, S_{n-1}(x)$ where each $S_k(x)$ is a straight line with equation

$$S_k(x) = a_k + b_k(x - x_k) \qquad \text{for } \; x_k \le x \le x_{k+1}, \qquad \text{with } k = 1, 2, \ldots, n-1.$$

Save the coefficients a_k, b_k in a matrix $\mathsf{S} = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ \vdots & \vdots \\ a_{n-1} & b_{n-1} \end{pmatrix}$ as `A12.dat`.

Evaluate the fitted curve at the points `linspace(0,24,100)` and save these values as a column vector in `A13.dat`.

Problem 2

In this problem you will construct a *quadratic* spline interpolant for a set of data points

$$(x_1, y_1),\ (x_2, y_2),\ (x_3, y_3),\ \ldots, (x_n, y_n)$$

Let $S(x)$ be the piecewise quadratic curve composed of $S_1(x), S_2(x), \ldots, S_{n-1}(x)$, where $S_k(x)$ denotes the spline on the kth subinterval $[x_k, x_{k+1}]$, for $k = 1, 2, \ldots, n-1$.
Suppose each $S_k(x)$ is a quadratic polynomial of the form

$$S_k(x) = a_k + b_k(x - x_k) + c_k(x - x_k)^2 \qquad \text{for } \; x_k \le x \le x_{k+1}.$$

Then there are $3(n-1)$ coefficients to determine. We impose the following conditions:

$\boxed{1}$ $S_k(x_k) = y_k$ for $k = 1, \ldots, n-1$ and $S_{n-1}(x_n) = y_n$
(the fitted curve $S(x)$ must go through all the data points: n conditions)

$\boxed{2}$ $S_k(x_{k+1}) = S_{k+1}(x_{k+1})$ for $k = 1, \ldots, n-2$
(adjacent splines join up at the interior knots: $n-2$ conditions)

$\boxed{3}$ $S'_k(x_{k+1}) = S'_{k+1}(x_{k+1})$ for $k = 1, \ldots, n-2$
(the gradients of adjacent splines match at the interior knots: $n-2$ conditions)

This makes a total of $3n-4$ conditions, whereas we have $3n-3$ unknown coefficients, so one extra condition is needed. This is usually imposed at the left boundary. Two possibilities are:

(i) gradient specified at the left endpoint: $S'_1(x_1) = \gamma$

(ii) zero second derivative at the left endpoint: $S''_1(x_1) = 0$

(PLEASE TURN OVER)

Follow the calculations we did in class for cubic splines to express the "c" coefficients in terms of the "b"s. Then derive an system of linear equations for the "b"s, and write it in the form

$$M\mathbf{b} = \mathbf{rhs} \qquad \text{where} \qquad \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_{n-2} \\ b_{n-1} \end{pmatrix}$$

For the data set `temp.txt`, do the following:

- Assuming boundary condition (i), save the associated $(n-1)\times(n-1)$ matrix M in `A21.dat`. Compute the coefficients a_k, b_k, c_k for $k = 1, \ldots, n-1$, put them in a matrix

  $$\mathsf{S} = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ \vdots & \vdots & \vdots \\ a_{n-1} & b_{n-1} & c_{n-1} \end{pmatrix} \qquad \text{and save this matrix as } \texttt{A22.dat}.$$

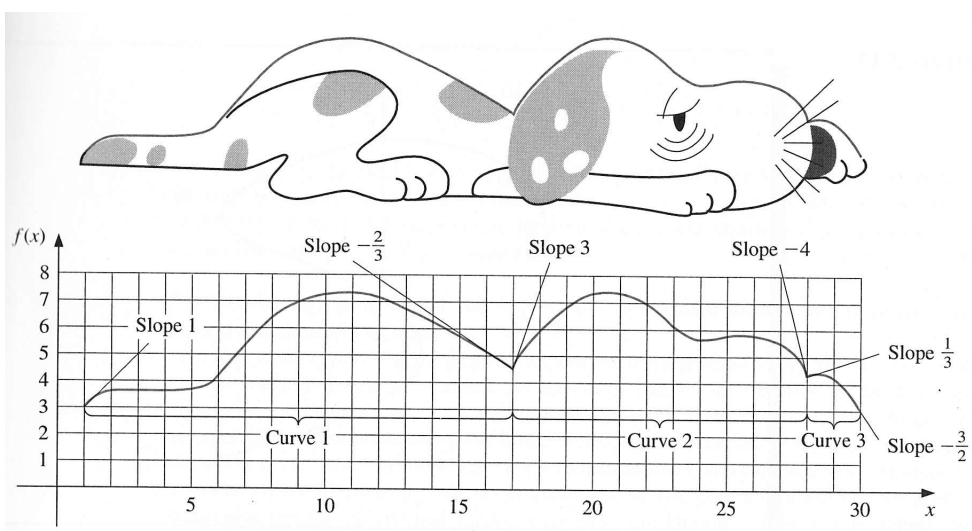
  Evaluate the fitted curve at the points `linspace(0,24,100)` and save these values as a column vector in `A23.dat`.

- Assuming boundary condition (ii), save the associated $(n-1)\times(n-1)$ matrix M, with 1 as the top left entry, in `A24.dat`. Compute the coefficients a_k, b_k, c_k for $k = 1, \ldots, n-1$, put them in an $(n-1)\times 3$ matrix S as above, and save this matrix as `A25.dat`.
  Evaluate the fitted curve at the points `linspace(0,24,100)` and save these values as a column vector in `A26.dat`.

Note: In your code you may want to use the trick of pretending that there is an nth spline $S_n(x)$ and thus solving an $n \times n$ linear system. If you do this, just make sure that the M and S you save have only $n-1$ rows.

Problem 3

The top portion of this picture of a dog is to be approximated by *three curves*, each of which is a *clamped cubic spline interpolant*. Download `dogtop.txt`, which contains a set of coordinates along the profile that we want to approximate. The point (17, 4.5) separates Curve 1 and Curve 2; the point (27.7, 4.1) separates Curve 2 and Curve 3. The slopes of each curve at its two endpoints are shown in the diagram.



Evaluate Curve 1 at the points `linspace(0,17,70)` and save these values as a column vector in `A31.dat`.
Evaluate Curve 2 at the points `linspace(17,27.7,50)` and save these values as a column vector in `A32.dat`.
Evaluate Curve 3 at the points `linspace(27.7,30,30)` and save these values as a column vector in `A33.dat`.