APMA 0160 (A. Yew) Spring 2011

Homework #4

Deadline: March 17 (Thursday) 4:00am EST
Scorelator will begin accepting submissions from March 10 (Thursday)

All the exercises in this assignment are to do with *least squares fitting*. For any problem that involves nonlinear curve fitting using Matlab's `fminsearch` command, *your initial guess of the parameter values is critical*! So even though the files you submit to Scorelator should not generate any actual graphical output, while working on the problems it is important to plot graphs to check whether you are on the right track.

Problem 1

Download the file `temp.dat` from the Homework section of the course webpage. This file contains temperature data recorded over a 24-hour period: the first column lists the number of hours after 12 noon of a certain day; the second column shows the temperature in degrees Fahrenheit measured at those times.

(a) Fit the data with the quadratic function

$$f(t) = at^2 + bt + c$$

Save the coefficients a, b and c (in that order) as a row vector in `A11.dat`.
Evaluate the fitted curve $f(t)$ at the points $t = 0 : 0.1 : 24$ (as if you were going to plot the parabola) and save these values as a column vector in `A12.dat`.
Calculate the root-mean-square error of this quadratic fit and save the result in `A13.dat`.

(b) Now fit the data with the trigonometric function

$$f(t) = a\cos(bt) + c$$

Save the coefficients a, b and c as a row vector in `A14.dat`.
Calculate the root-mean-square error of this cosine fit and save the result in `A15.dat`.

Problem 2

Download the file `decay.dat`, which contains a set of data points with the first column being x-values and the second column being the corresponding y-values.

(a) Fit the data with the exponential function

$$f(x) = ce^{ax}$$

Save the coefficients a and c (in that order) as a row vector in `A21.dat`.
Evaluate the fitted curve at the points `linspace(0,2,100)` (as if you were going to plot the curve) and save these values as a column vector in `A22.dat`.
Calculate the root-mean-square error of this exponential fit and save the result in `A23.dat`.

(b) It has been suggested that using a sum of two exponentials would produce a better fit. Therefore, now try to fit the data with

$$f(x) = c_1e^{a_1x} + c_2e^{a_2x}$$

Save the coefficients c_1, a_1, c_2 and a_2 (in that order) as a row vector in `A24.dat`.
Calculate the root-mean-square error of this double-exponential fit and save the result in `A25.dat`.

(please turn over)

Problem 3

Download the file `yeast.dat`, which contains data collected by the biologist T. Carlson in 1913 on the growth of a population of yeast cells: the first column lists the number of days after data collection began; the second column shows the biomass of the yeast culture (a measure of the population size) recorded on those days.

It has been suggested that the data is well described by "logistic growth"; that is, the population size grows according to the logistic differential equation

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

where the parameter r represents the initial per capita growth rate and K is the so-called environmental carrying capacity. The solution of this differential equation can be expressed explicitly as

$$P(t) = \frac{1}{\frac{1}{K} + ce^{-rt}}$$

Therefore, fit the data with the function

$$f(t) = \frac{1}{\frac{1}{K} + ce^{-rt}}$$

Save the coefficients K, c and r (in that order) as a row vector in `A31.dat`.
Evaluate the fitted curve at the points $t = 0 : 0.2 : 20$ and save these values as a column vector in `A32.dat`.
Calculate the root-mean-square error of this logistic fit and save the result in `A33.dat`.