APMA 0160 (A. Yew) Spring 2011

Homework #3

Deadline: March 10 (Thursday) 4:00am EST
Scorelator will begin accepting submissions from March 3 (Thursday)

Problem 1

Consider the linear system $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{pmatrix} 3 & 3+10^{-13} & 1.7 & -5 \\ 2 & 2 & 21 & 8 \\ 3 & 6 & 4 & -10 \\ -1 & 1 & 8.6 & 7 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 2.7 \\ 33 \\ 3 \\ 15.6 \end{pmatrix}$$

(a) Find the LU factorization of A using step-by-step Gaussian elimination without any row pivoting. Save the lower triangular matrix in `A11.dat` and the upper triangular matrix in `A12.dat`.

(b) Using the factorization you found in part (a), solve $A\mathbf{x} = \mathbf{b}$ by forward and backward substitution. Save the solution as a column vector in `A13.dat`.

(c) Now solve $A\mathbf{x} = \mathbf{b}$ by LU factorization with partial pivoting (you may use the built-in Matlab command). Save the solution of $L\mathbf{y} = P\mathbf{b}$ in `A14.dat` and the solution of $U\mathbf{x} = \mathbf{y}$ in `A15.dat`.

Problem 2

Recall the system of equations

$$\begin{aligned} R_6 I_1 + R_1(I_1 - I_2) + R_2(I_1 - I_3) &= V_1 \\ R_3 I_2 + R_4(I_2 - I_3) + R_1(I_2 - I_1) &= V_2 \\ R_5 I_3 + R_4(I_3 - I_2) + R_2(I_3 - I_1) &= V_3 \end{aligned}$$

from Homework #2, where the resistances are $R_1 = 20$, $R_2 = 10$, $R_3 = 25$, $R_4 = 10$, $R_5 = 30$, $R_6 = 40$ and the voltages are $V_2 = 0$ and $V_3 = 200$, while V_1 is variable. The currents I_1, I_2 and I_3 are the unknowns. In this form, the coefficient matrix is strictly diagonal dominant by row.

(a) Increase V_1 from 0 to 100 in steps of 4 (i.e. take $V_1 = 0,\ 4,\ 8, \ldots, 96,\ 100$) and for each V_1 value calculate I_1, I_2 and I_3 by solving the system with the **Jacobi iteration scheme**. Begin with the initial guess $(I_1, I_2, I_3) = (0, 0, 0)$; for the stopping criterion, require that the 2-norm of the difference between successive iterates be no more than 10^{-6}. Save your results in `A21.dat` as a 3×26 matrix, where the first column contains the I_1, I_2, I_3 values corresponding to $V_1 = 0$, the second column contains the I_1, I_2, I_3 values corresponding to $V_1 = 4$, and so on. Also, for each V_1 value record the number of iterations needed to solve the linear system to the given accuracy, and save these numbers as a length-26 row vector in `A22.dat`.

(b) Repeat part (a), but this time with the **Gauss–Seidel iteration scheme**. Use the same initial guess and the same stopping criterion, and save the solutions (I_1, I_2, I_3) for the increasing V_1 values as a 3×26 matrix in `A23.dat`. Save the number of iterations needed at each V_1 value as a length-26 row vector in `A24.dat`.

(c) Finally, based on the data in `A22.dat` and `A24.dat`, calculate the *average* number of iterations needed by each of the Jacobi and Gauss–Seidel methods (find a suitable Matlab command or commands to do this). Save these numbers in `A25.dat` as a row vector with two components (Jacobi first and Gauss–Seidel second).