APMA 0160 (A. Yew) Spring 2011

Homework #10

Deadline: May 7 (Saturday) 4:00am EST

Problem 1

Consider the Van der Pol differential equation

$$(\star) \qquad \frac{d^2y}{dx^2} + \mu(y^2-1)\frac{dy}{dx} + y = 0 \qquad \text{for } 0 \le x \le 2$$

with $\mu = -\frac{1}{2}$, subject to the boundary conditions

$$(\dagger) \qquad y(0) = 0, \qquad y(2) = 1$$

(a) (i) Use `ode45` with default tolerances to solve the initial value problem consisting of the ODE $(\star)$ and the initial conditions $y(0) = 0$, $y'(0) = 1$. Find the value of the computed solution at $x = 2$.
(ii) Use `ode45` with default tolerances to solve the initial value problem consisting of the ODE $(\star)$ and the initial conditions $y(0) = 0$, $y'(0) = 2$. Find the value of the computed solution at $x = 2$.
Save the values at $x = 2$ of the computed solutions from (i) and (ii) as a row vector in `A11.dat`.

(b) Use the shooting method to find an approximate solution of the boundary value problem consisting of the ODE $(\star)$ and the boundary conditions $(\dagger)$. Use `ode45` with default tolerances for "time"-stepping, and use the information from part (a) together with the bisection method for root-finding. Stop iterating when the value of the computed solution at $x = 2$ gets within 10^{-8} of the target value. Then save the number of iterations and the slope at the left boundary as a row vector in `A12.dat`, and save the y values of the computed solution at $x = 0 : 0.01 : 2$ as a column vector in `A13.dat`.

Problem 2

Consider the differential equation

$$(*) \qquad y'' = \frac{1}{8}\left(32 + 2x^3 - yy'\right) \qquad \text{for } 1 \le x \le 3$$

subject to the boundary conditions

$$(\ddagger) \qquad y(1) = 17, \qquad y'(3) = 0$$

(note that the condition at the right endpoint involves the *derivative* of y).

(a) (i) Use `ode45` with default tolerances to solve the initial value problem consisting of the ODE $(*)$ and the initial conditions $y(1) = 17$, $y'(1) = 0$. Find the slope of the computed solution at $x = 3$.
(ii) Use `ode45` with default tolerances to solve the initial value problem consisting of the ODE $(*)$ and the initial conditions $y(1) = 17$, $y'(1) = -40$. Find the slope of the computed solution at $x = 3$.
Save the slopes at $x = 3$ of the computed solutions from (i) and (ii) as a row vector in `A21.dat`.

(b) Use the shooting method to find an approximate solution of the boundary value problem consisting of the ODE $(*)$ and the boundary conditions $(\ddagger)$. Use `ode45` with default tolerances for "time"-stepping, and use the information from part (a) together with the bisection method for root-finding. Stop iterating when the slope of the computed solution at $x = 3$ gets within 10^{-10} of the target value. Then save the number of iterations and the slope at the left boundary as a row vector in `A22.dat`, and save the y values of the computed solution at $x = 1 : 0.02 : 3$ as a column vector in `A23.dat`.

(PLEASE TURN OVER)

Problem 3

Download `velocity.txt` from the Homework section of the course webpage (the same file that was used in Homework #7). This file contains data on the velocity of an object (in meters per second) measured at a sequence of times (seconds). The first column lists the times and the second column the corresponding velocities.

Compute the approximate acceleration (rate of change of velocity) as a function of time. Do this in the following ways:

(a) Apply a finite difference numerical differentiation scheme to the raw data. Use an $O(\Delta t^2)$ centered difference formula for the interior points, together with $O(\Delta t)$ forward and backward difference formulas for the boundary points. Save the results (a sequence of approximate accelerations at the data points) as a column vector in `A31.dat`.

(b) Fit a clamped cubic spline interpolant through the data points, with slopes at the left and right endpoints set equal to the $O(\Delta t)$ forward and backward difference quotients at the boundary of the data set. Then evaluate the fitted curve at $t = 0 : 0.1 : 30$ and apply a finite difference numerical differentiation scheme to this set of points.

(i) Use an $O(\Delta t^2)$ centered difference formula for the interior points, together with $O(\Delta t^2)$ forward and backward difference formulas at the boundaries. Save the results (a sequence of approximate accelerations at $t = 0 : 0.1 : 30$) as a column vector in `A32.dat`.

(ii) Use an $O(\Delta t^4)$ centered difference formula for the interior points, together with $O(\Delta t^2)$ formulas at the boundaries—use forward and backward differences at $t = 0$ and $t = 30$, and centered differences at $t = 0.1$ and $t = 29.9$. Save the results (a sequence of approximate accelerations at $t = 0 : 0.1 : 30$) as a column vector in `A33.dat`.