
Iterated transition matrices

Example 1
A survey by a state's utility commission has revealed information about the probabilities of conversion between different energy sources for home heating. The principal energy sources are gas (G), oil (O) and electricity (E). The data from the study indicates the percentage of households that are likely to convert from one fuel source to another in a given year; this information is summarized in matrix form as follows:

$$\mathbf{H} = \begin{array}{c} \text{Currently have} \\ \begin{array}{ccc} \text{G} & \text{O} & \text{E} \end{array} \\ \begin{pmatrix} 0.919 & 0.175 & 0 \\ 0.060 & 0.825 & 0.049 \\ 0.021 & 0 & 0.951 \end{pmatrix} \end{array} \begin{array}{l} \text{G} \\ \text{O} \\ \text{E} \end{array} \quad \begin{array}{l} \text{Will} \\ \text{switch} \\ \text{to} \end{array}$$

Suppose that a certain county in the state has 100,000 households using gas, 70,000 using oil, and 30,000 using electricity. Predict the number of households using each type of fuel after one year, five years, ten years, and twenty years. Do these numbers seem to be settling down?

Find the steady state distribution of the number of households using each type of fuel.

Example 2
An environmental monitoring agency measures air quality with respect to pollution using the following rating scale:

C — clear: low pollution levels

M — moderate levels of pollution

W — warning: people with respiratory problems are advised to take precautions

A — alert: stagnated air, dangerous levels of pollution

For a certain city, the probabilities of transition between the four air-quality levels from one day to the next are summarized in a matrix:

$$\mathbf{T} = \begin{array}{c} \text{Current air quality} \\ \begin{array}{cccc} \text{C} & \text{M} & \text{W} & \text{A} \end{array} \\ \begin{pmatrix} 0.4 & 0.3 & 0.2 & 0 \\ 0.3 & 0.3 & 0.3 & 0.2 \\ 0.2 & 0.2 & 0.3 & 0.5 \\ 0.1 & 0.2 & 0.2 & 0.3 \end{pmatrix} \end{array} \begin{array}{l} \text{C} \\ \text{M} \\ \text{W} \\ \text{A} \end{array} \quad \begin{array}{l} \\ \text{Air quality} \\ \text{the next day} \\ \\ \end{array}$$

If there is moderate air pollution in the city today, predict the distribution of probabilities of the different levels of pollution ten days from now.

Find the steady state distribution of probabilities for the four different levels of pollution.