

This system of equations can be written in matrix form $A\mathbf{y} = \mathbf{rhs}$ as follows:

$$\begin{pmatrix}
 2 + q(x_1)\Delta x^2 & -1 + \frac{p(x_1)}{2}\Delta x & 0 & \dots & \dots & \dots & \dots & 0 \\
 -1 - \frac{p(x_2)}{2}\Delta x & 2 + q(x_2)\Delta x^2 & -1 + \frac{p(x_2)}{2}\Delta x & 0 & \dots & \dots & \dots & 0 \\
 0 & -1 - \frac{p(x_3)}{2}\Delta x & 2 + q(x_3)\Delta x^2 & -1 + \frac{p(x_3)}{2}\Delta x & 0 & \dots & \dots & 0 \\
 0 & 0 & & & & & & \vdots \\
 \vdots & \vdots & & & & & & \vdots \\
 & & & & & & & 0 \\
 0 & 0 & 0 & \dots & 0 & -1 - \frac{p(x_{N-1})}{2}\Delta x & 2 + q(x_{N-1})\Delta x^2 & -1 + \frac{p(x_{N-1})}{2}\Delta x \\
 0 & \dots & \dots & \dots & \dots & 0 & -1 - \frac{p(x_N)}{2}\Delta x & 2 + q(x_N)\Delta x^2
 \end{pmatrix}
 \begin{pmatrix}
 y_1 \\
 y_2 \\
 y_3 \\
 y_4 \\
 \vdots \\
 \vdots \\
 y_{N-2} \\
 y_{N-1} \\
 y_N
 \end{pmatrix}
 =
 \begin{pmatrix}
 -r(x_1)\Delta x^2 + \left(1 + \frac{p(x_1)}{2}\Delta x\right)\alpha \\
 -r(x_2)\Delta x^2 \\
 -r(x_3)\Delta x^2 \\
 \vdots \\
 \vdots \\
 -r(x_{N-1})\Delta x^2 \\
 -r(x_N)\Delta x^2 + \left(1 - \frac{p(x_N)}{2}\Delta x\right)\beta
 \end{pmatrix}$$

A is an $N \times N$ triadiagonal matrix; $\mathbf{y}$ is the vector of y values that we want to find.

The boundary conditions appear in the first and last rows of $\mathbf{rhs}$.

To solve for $\mathbf{y}$, we can use `A\rhs` in Matlab.