“Takeaway” Games!

Steven Kim

$20 game

- Paper clips attached to this $20 bill.

- Rules:

    1. We alternate turns;

    2. On your turn, take **1, 2, or 3** clips.

- **Goal:** take the last paper clip.

Let’s play!

- Play with a partner by taking from a shared pile of paper clips;

- (but no money).

How did you do?

- Who won? The player who went first or the player who went second?

- *Does it matter?*

- **Play again,** but this time pick carefully whether you want to go <u>first</u> or <u>second</u>!

How can you win??

- On **my** turn: if only **1** or **2** or **3** clips remain to take from, then I win!
- *(What if 4 clips remain?)*

- So at the end of my turn (beginning of your turn): I want to **leave you with 4 clips remaining.**

- **Goal**: reach **0** clips … by reaching **4** clips … by reaching **8** clips … by reaching **12** clips … etc.

This is an old game!

- The $20 game is a simple version of a game called **Nim**.
- The *Nimatron* machine played Nim (against humans) at the 1940 NY World Fair.
- Maybe originated in 16th century Europe, maybe earlier from China.

Why is this math?

- Because we add numbers? Sort of, but there's more!
- Mathematicians try to **simplify**:
    - when a problem is "hard", we make it simpler until we can understand;
    - in this case, imagine starting with fewer clips.
- Mathematicians also try to **generalize**:
    1. what is the **main idea**?
    2. what other games share a **similar** main idea?

Generalize?

- The *main idea* is to “take away” turn-by-turn.
- What if you have **30 (or more!)** clips?
- What if you can take away **1, 2, 3, 4, 5 (or more!)** each turn, instead of just 1, 2, 3?
- What if you can only take away clips of **same color**?
- What if you **lose** by taking the last one?
- Instead of taking clips, imagine **taking space**?

“Taking space” game: **Sprouts**!

1. start with **n** dots;

2. each turn, **if possible**:

   A. draw a curve from one dot to another, or to itself;

   B. draw a new dot on that new curve;

3. a curve **cannot intersect** any curve/dot;

4. dots allowed to have **max of 3** lines going out (curve from dot to itself counts as two); it helps to draw a circle around such dots;

5. if no move is possible, you lose!

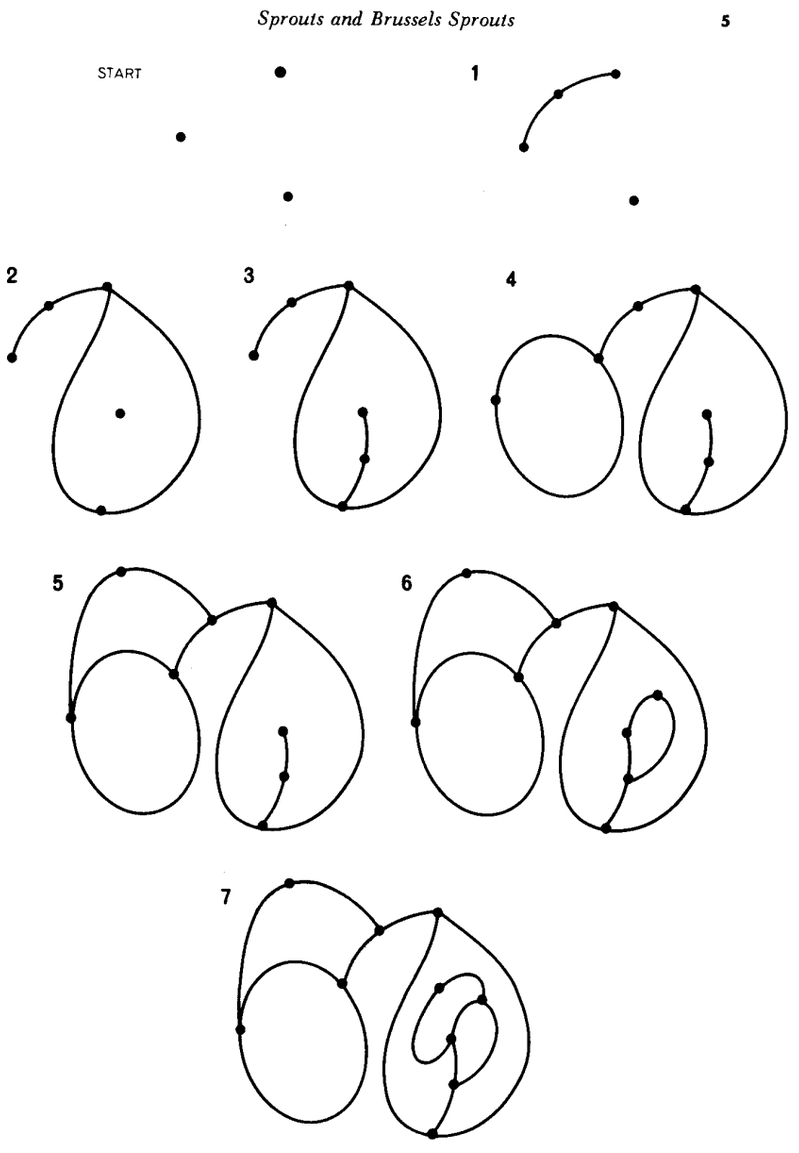


FIGURE 1
A typical game of three-spot sprouts

Let’s play!

- Play with a partner on a piece of paper!

What's the math?

- Why is there **"not enough space"** to draw more?
- Is it better to be **first or second** player?
- What's the **longest** game possible?
- What's the **shortest** game possible?

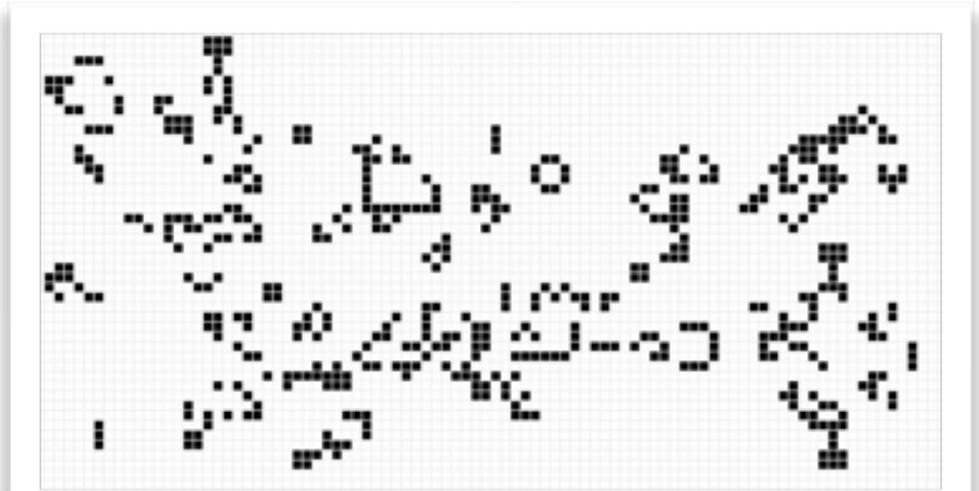
What do we know?

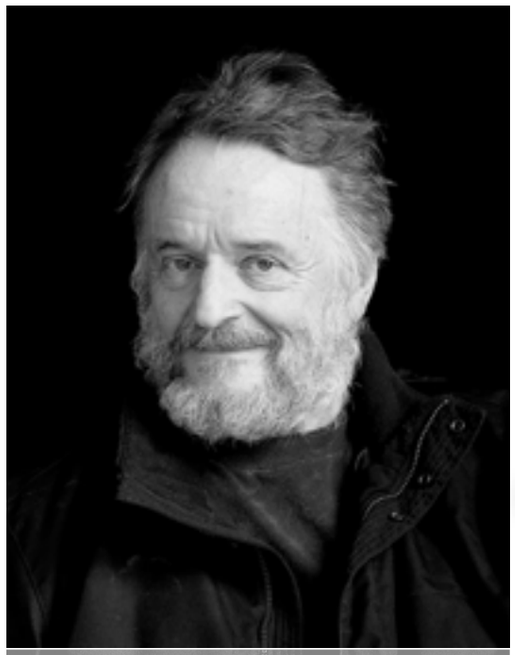
Each dot has 3 “lives” (curves that can come out).
Each move *kills* two lives (by drawing a curve) but *makes* one life (by drawing a new dot).

- **longest game?** 3n - 1 moves.
- **shortest game?** 2n moves.
- **go first or second?** depends on n!!
- **not enough space??**

Not just fun and games!

- Mathematicians ask: *how do things change over time?*
- Games ask: *how do things change due to players' actions?*
- In games, math, and the real world: simple rules lead to complex behavior!
- *Sprouts* created by **John Conway;** he really likes games





For the advanced crowd: Brussels Sprouts

1. start with **n** crosses (+)

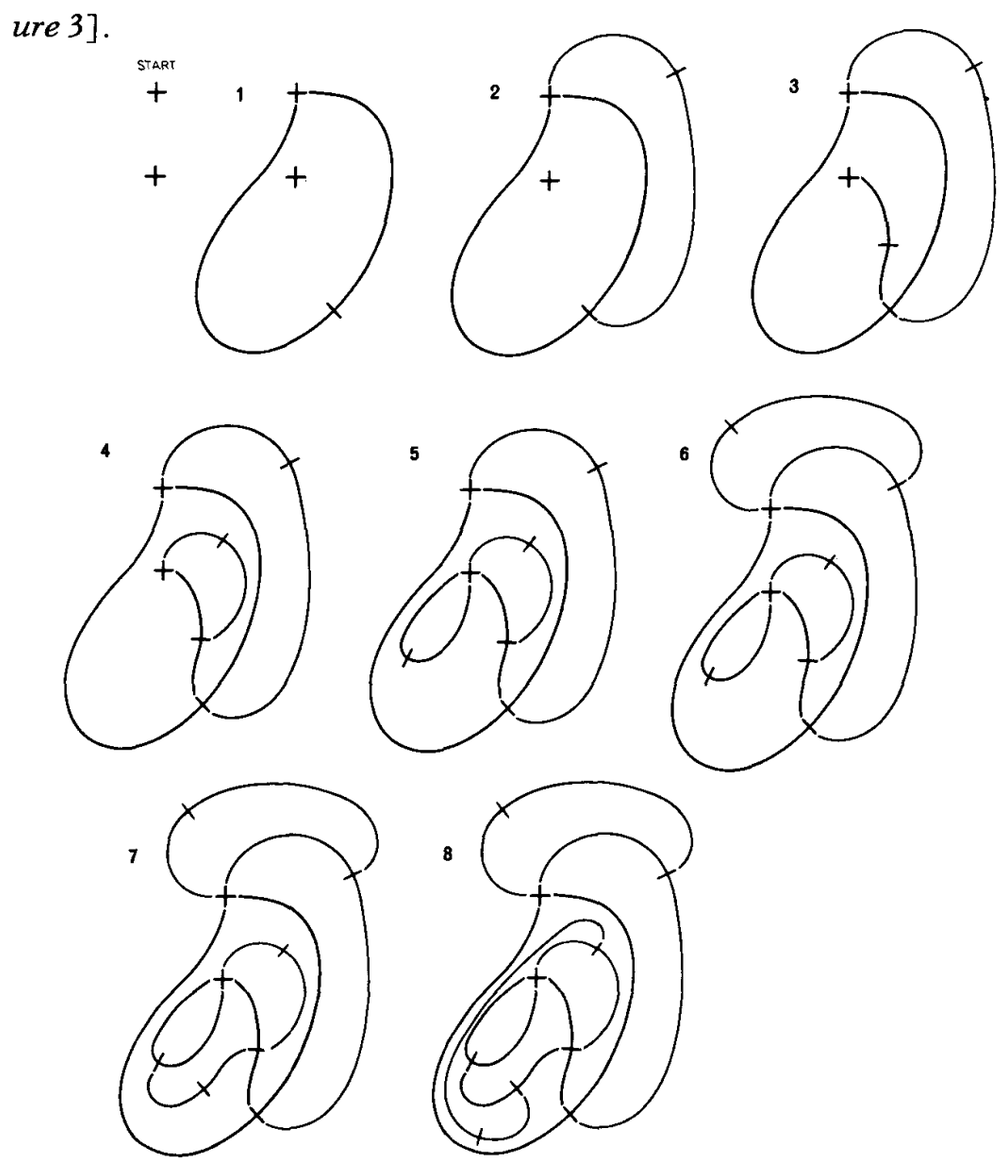
2. each turn,

   A. draw a curve connecting two unused arms (of the crosses)

   B. draw a bar crossing that new curve (forms two new arms)

3. a curve cannot intersect anything else

4. if you can't make a move, you lose



Math says this game is **boring**!

- If **m** = # of turns,
  - #crosses = n + m (one new cross made each turn)
  - #segments = 2m (two new segments made each turn)
  - #regions = 4n (one free arm per region, each move kills 2 arms and makes 2 arms, and game starts with 4n arms)
- Fact: **2 = #regions - #segments + #crosses** = 4n-2m+n+m
- So **m = 5n - 2** always.
- The "Fact" is called the **Euler characteristic.** To show it is true, start with only a few crosses, check the answer, and then add more crosses one at a time.

For more:

http://en.wikipedia.org/wiki/Nim

http://en.wikipedia.org/wiki/Sprouts_(game)

Martin Gardner,
“Mathematical Magic Show”