Monty Hall Problem

Let's Make a Deal!

Monty Hall

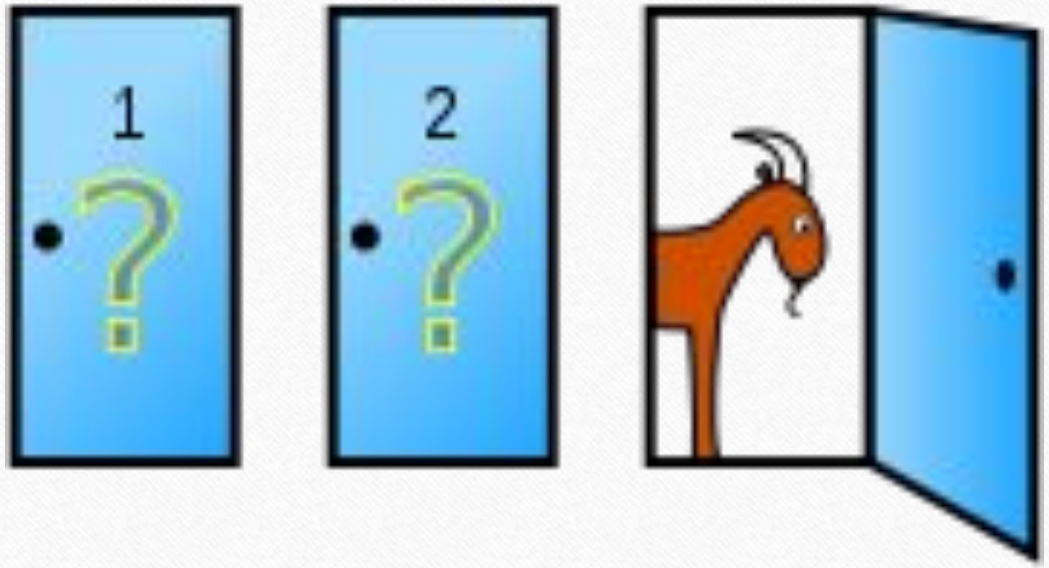
Movie "21":

https://www.youtube.com/watch?v=Zr_xWfThjJ0

Reader's letter to **Ask Marilyn** column in the *Parade* magazine in 1990:

> Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats.
>
> You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

Your pick



Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats.

You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

Take 1-2 minutes to discuss this with the person sitting next to you.

Marilyn vos Savant's response was that the contestant should switch to the other door.

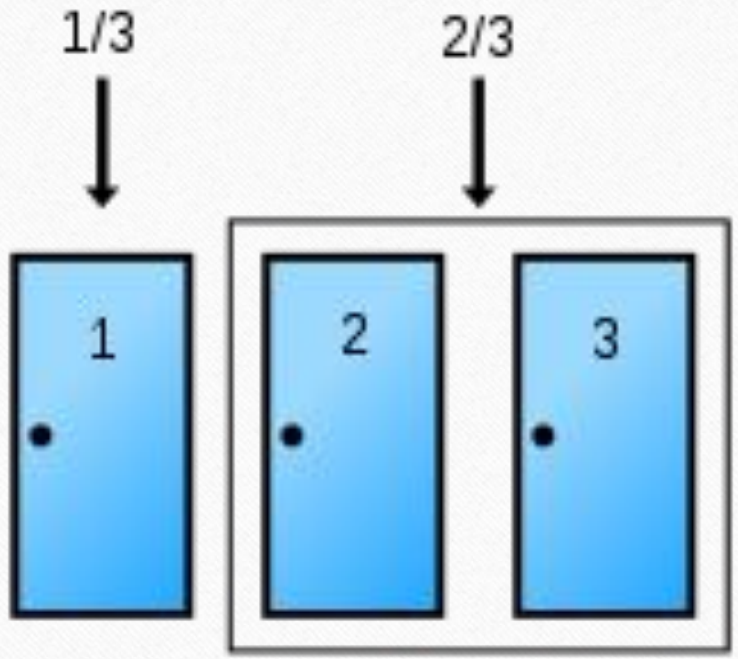
Many readers thought that she was wrong, and she received many letters from angry readers.

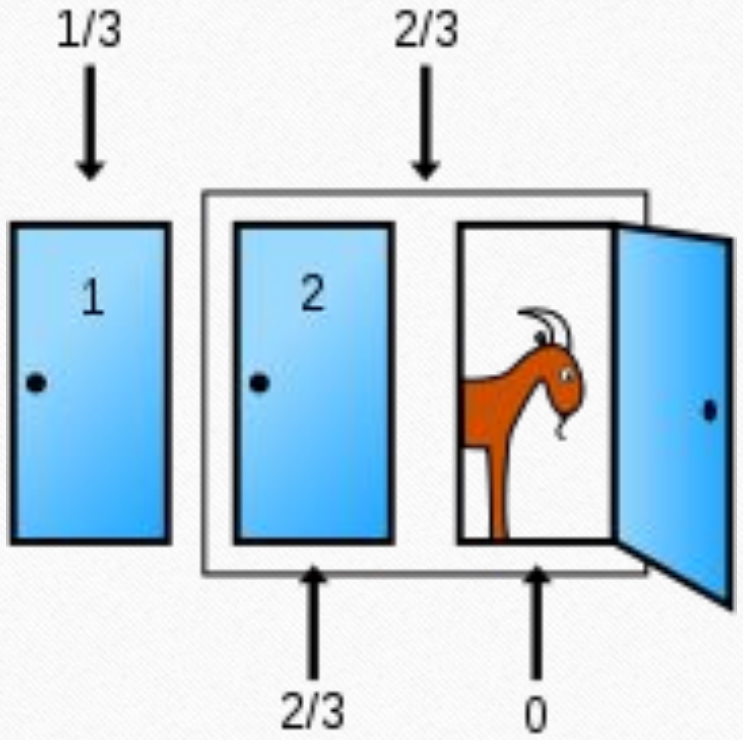
"You blew it, and you blew it big! Since you seem to have difficulty grasping the basic principle at work here, I'll explain. After the host reveals a goat, you now have a one-in-two chance of being correct. Whether you change your selection or not, the odds are the same. There is enough mathematical illiteracy in this country, and we don't need the world's highest IQ propagating more. Shame!"
– Scott Smith, Ph.D. University of Florida
(vos Savant 1990a)

She was right! How?

Intuitively, her solution:

Behind door 1	Behind door 2	Behind door 3	Result if staying at door #1	Result if switching to the door offered
Car	Goat	Goat	**Wins car**	Wins goat
Goat	**Car**	Goat	Wins goat	**Wins car**
Goat	Goat	**Car**	Wins goat	**Wins car**

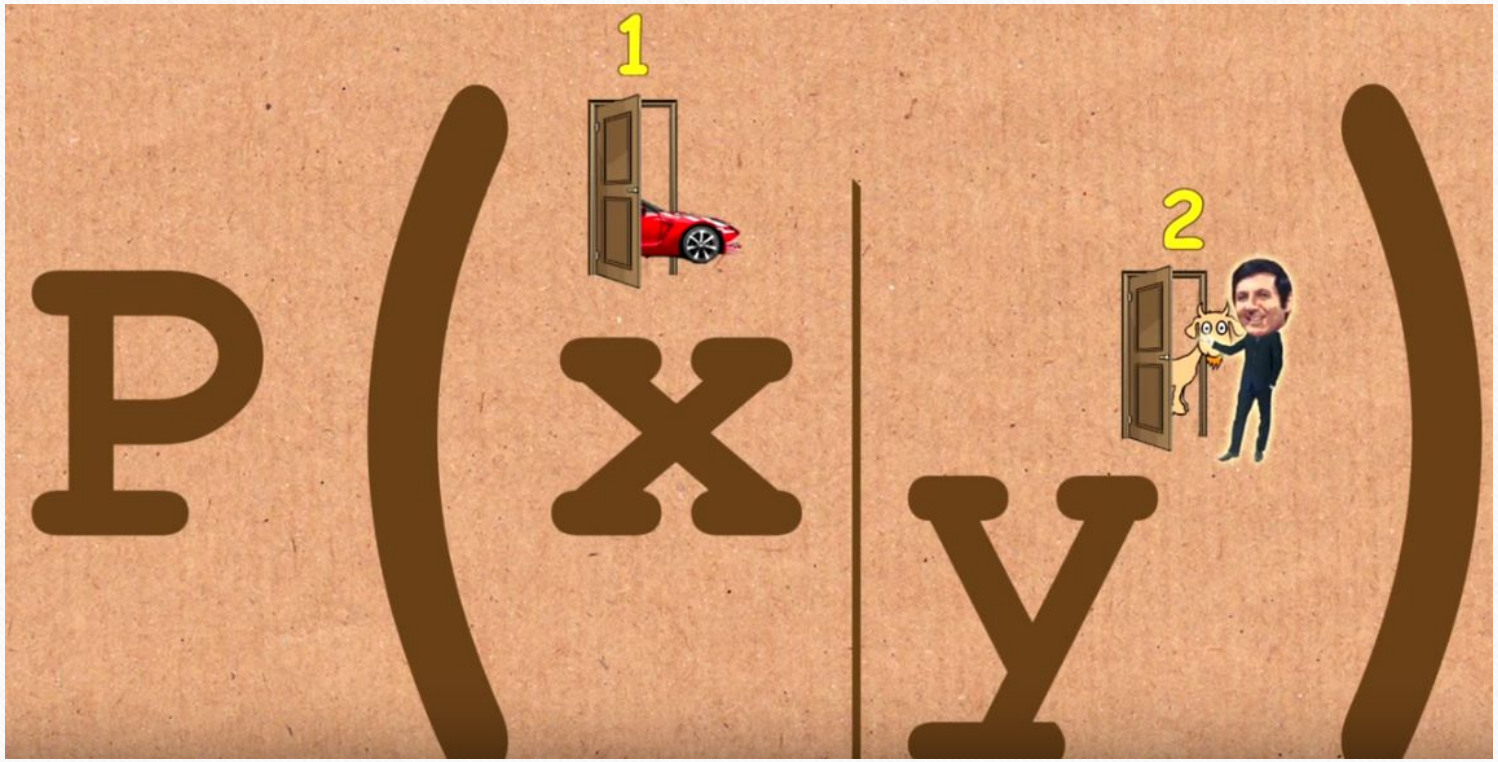




Note: extra info from Monty very important!

Solution using Conditional Probabilities

Takes into account extra info from Monty:



$$P(1 \,\&\, 2) =$$

$$P(1 \mid 2)\,P(2)$$

Use symmetry:

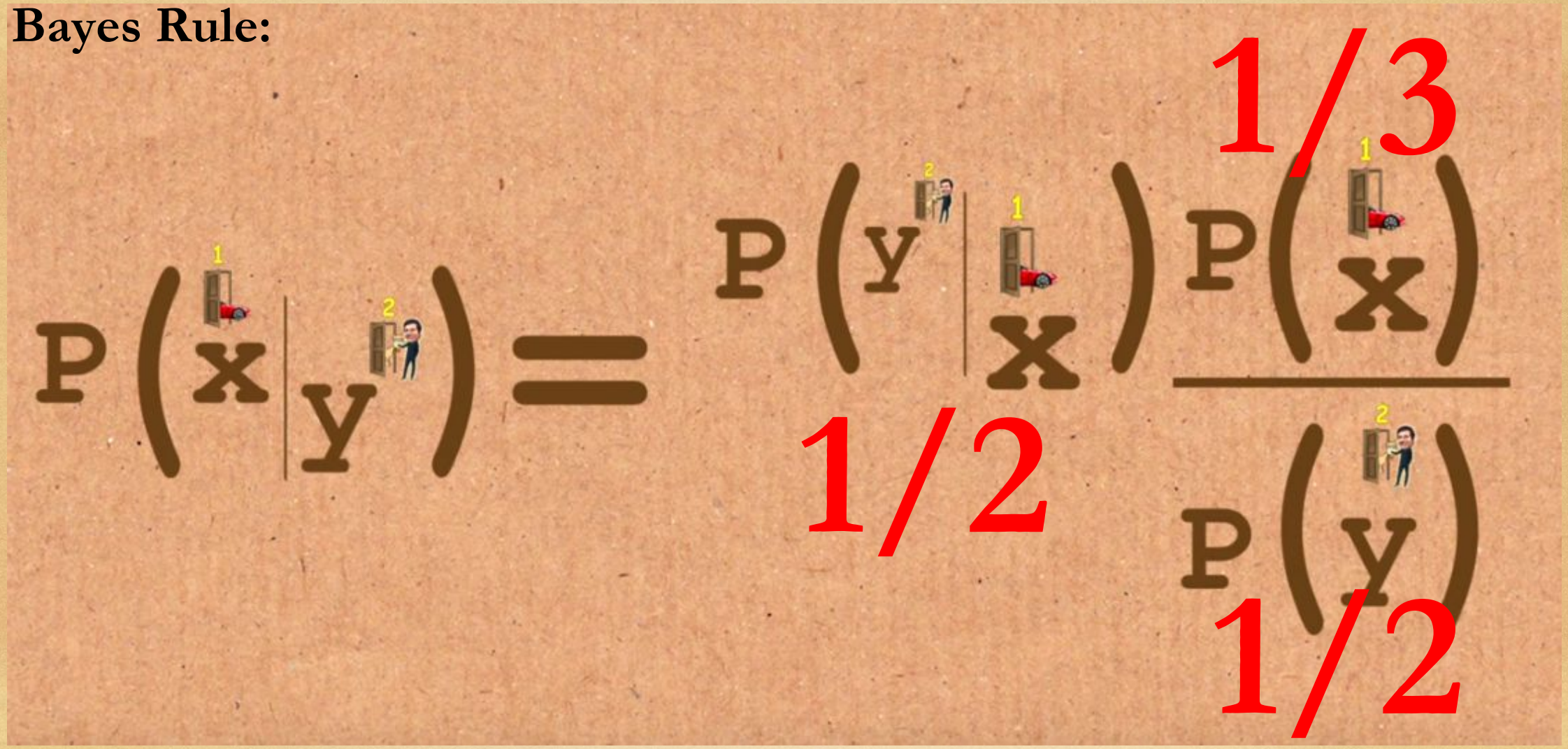
$$P(x_1 \& y_2) = P(x_1 \mid y_2)\,P(y_2)$$

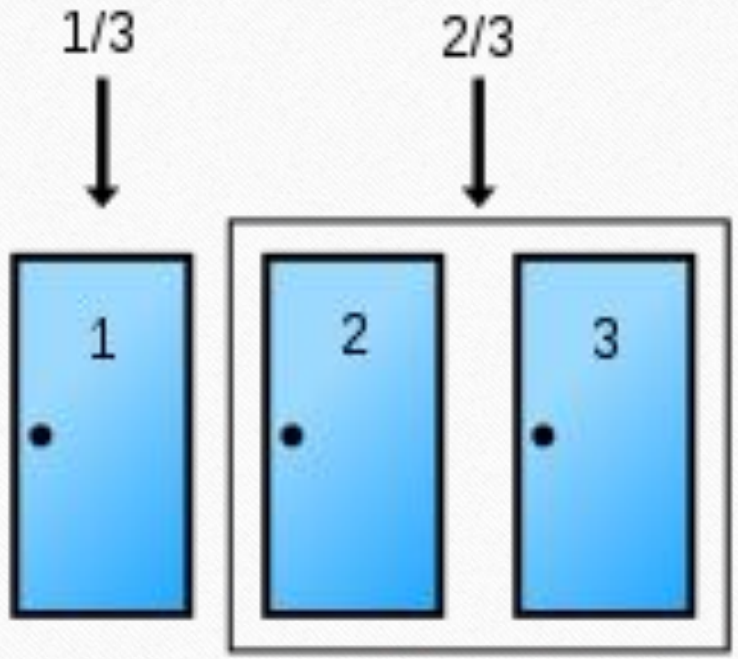
$$P(y_2 \& x_1) = P(y_2 \mid x_1)\,P(x_1)$$

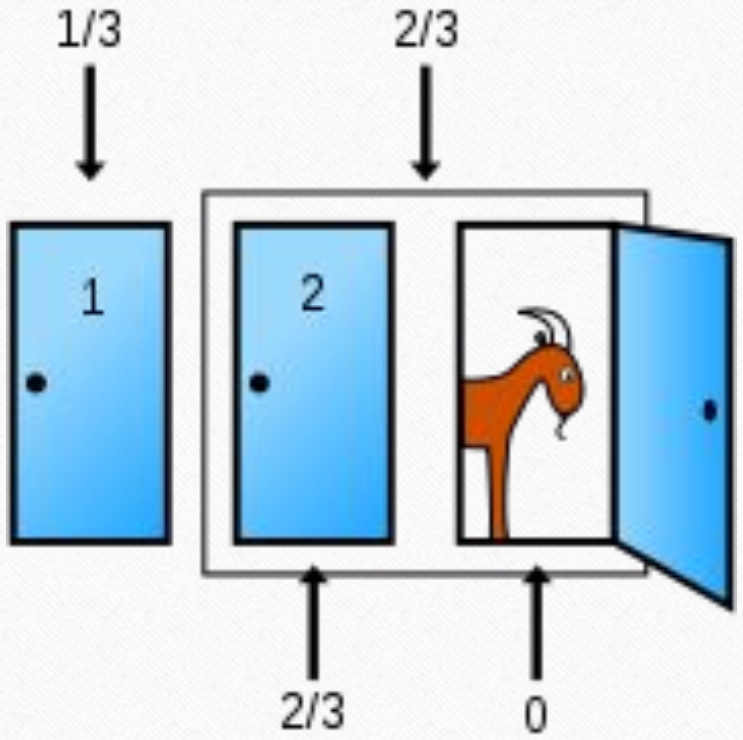
Bayes' Rule:

$$P(x_1 \mid y_2) = \frac{P(y_2 \mid x_1)\,P(x_1)}{P(y_2)}$$

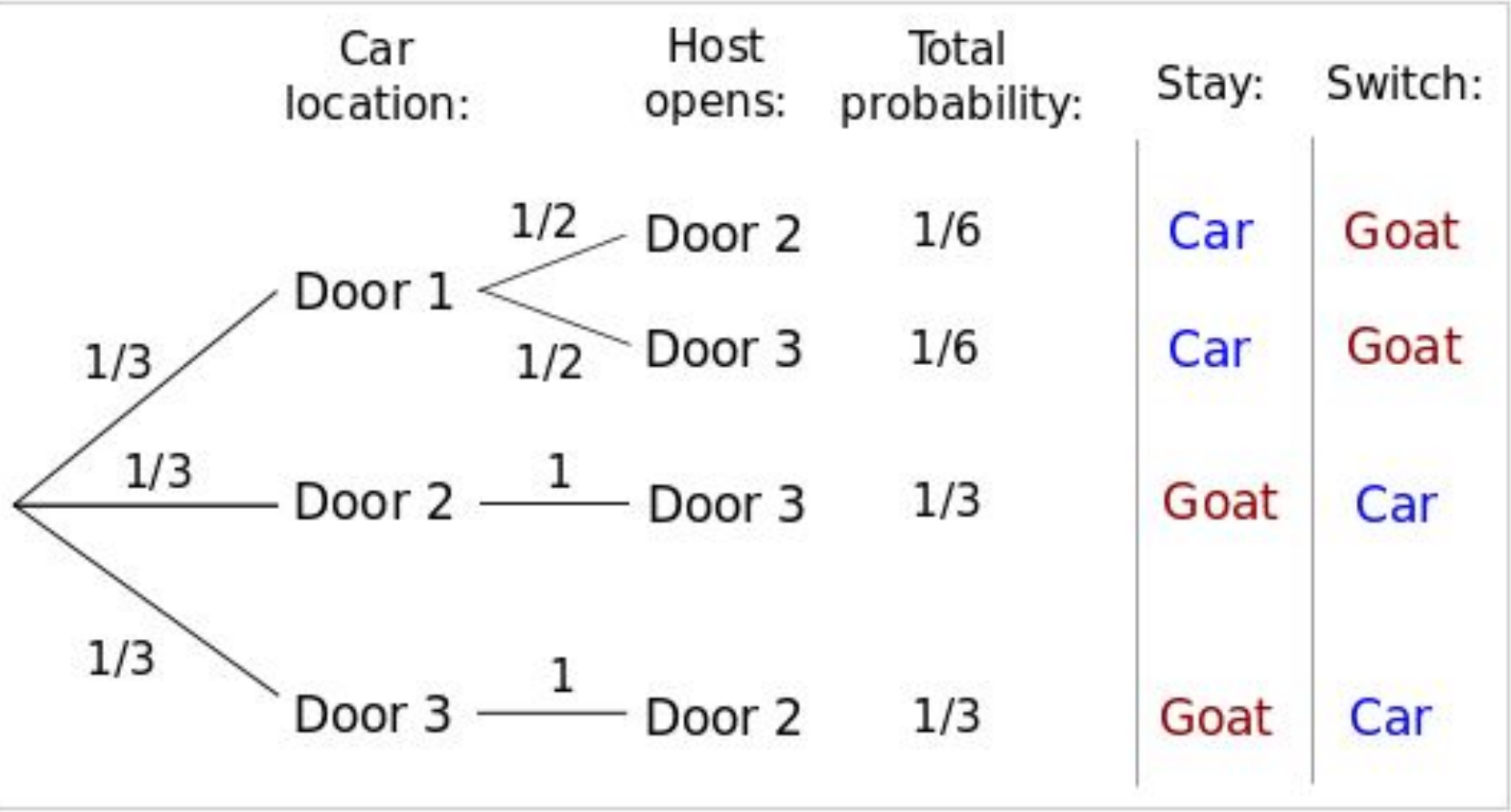
Bayes Rule:




1/3
2/3
1
2
3

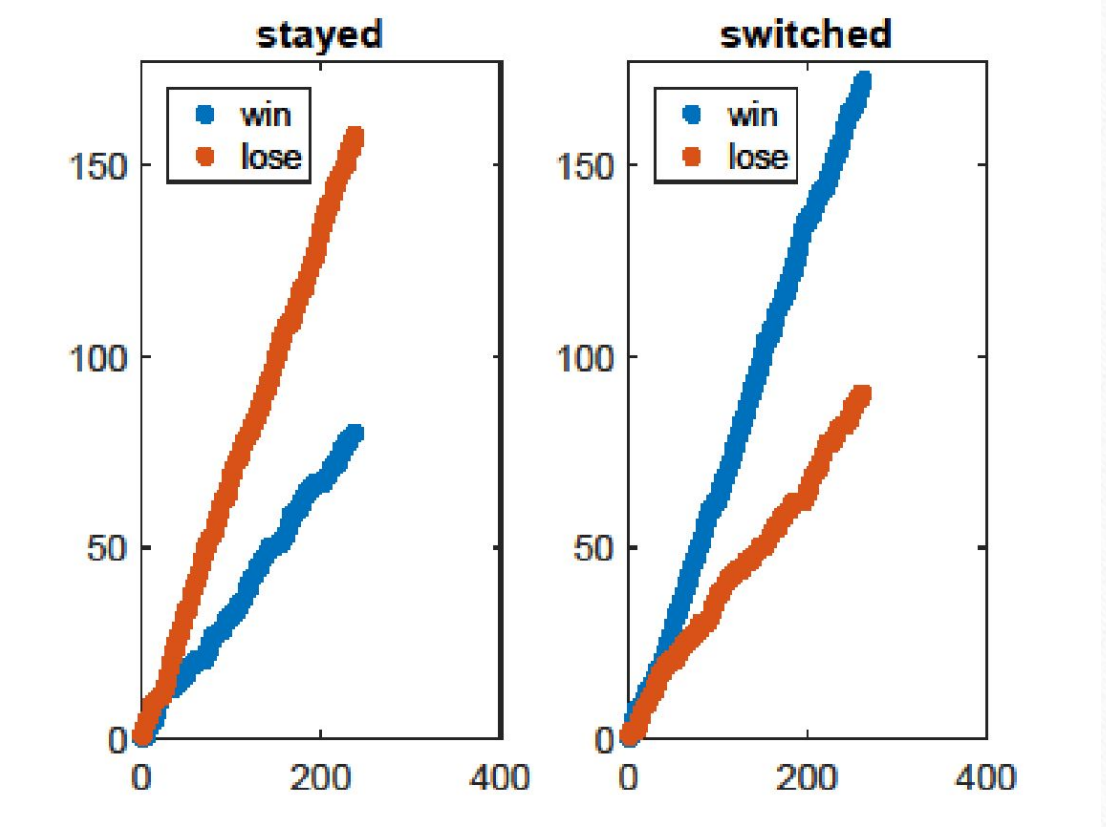

1/3
2/3
1
2
2/3
0

Probability tree



Tree showing the probability of every possible outcome if the player initially picks Door 1

People have also validated this numerically (see Matlab code)



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