Games of Chance

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Mathematics of Chance

- Experiment: A situation involving chance
- Event: The result of an experiment
- Probability: The chance of an event in an experiment

Let’s roll some dice

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Probability
# of 1’s						
# of 2’s						
# of 3’s						
# of 4’s						
# of 5’s						
# of 6’s						

Let’s roll 2 dice

Dice 1 / Dice 2	# of 1’s	# of 2’s	# of 3’s	# of 4’s	# of 5’s	# of 6’s
# of 1’s						
# of 2’s						
# of 3’s						
# of 4’s						
# of 5’s						
# of 6’s						

- Sample Space: Set/Collection of all possible outcomes
- What did we compute?
- Experimental probability: The chance of an event that actually occurs in an experiment
- Theoretical probability: The chance that it should happen

What is the difference between theoretical and experimental probabilities?

Law of Large Numbers

- Average of experimental probability obtained from a large number of (independent) trials gets closer to the average of the theoretical probability

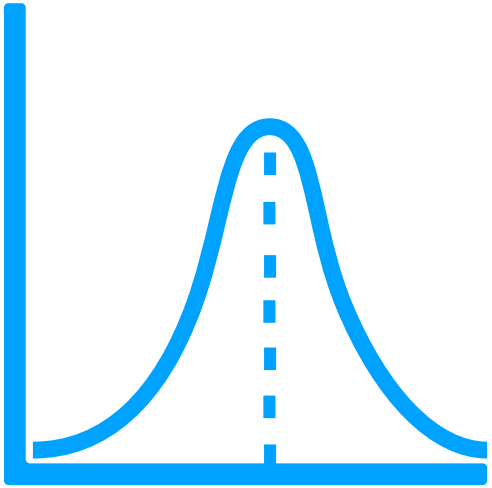
- Proved by Kolmogorov in 1930

A. Kolmogorov
(1903-1987)

Central Limit Theorem

- Histogram of the probabilities of the sum of many (independent) events has a bell-shaped curve

- Also known as a Gaussian

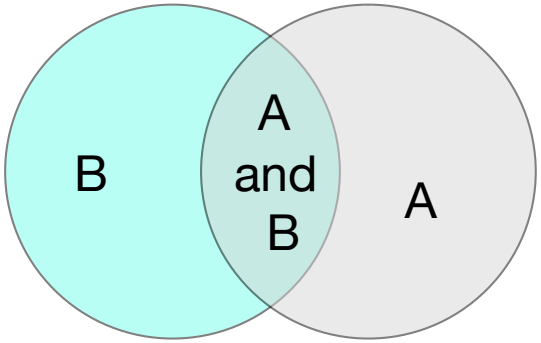


Carl Gauss
(1777-1885)

Some rules

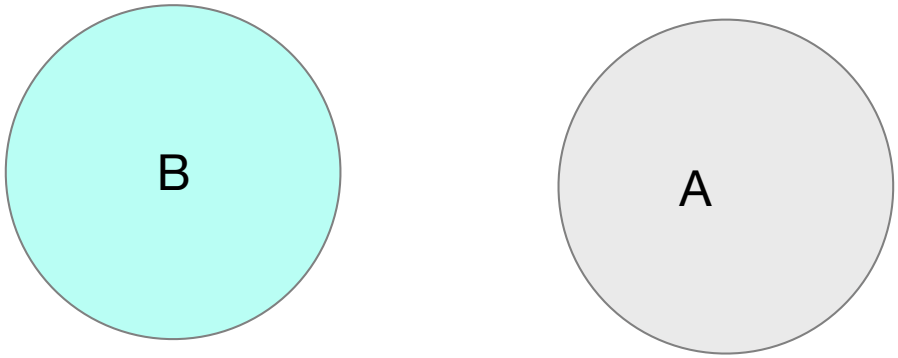
- Probability lies between 0 and 1
- Total probability of (all outcomes of) an experiment =1
- Denote the probability of any event A by P(A)

Some rules



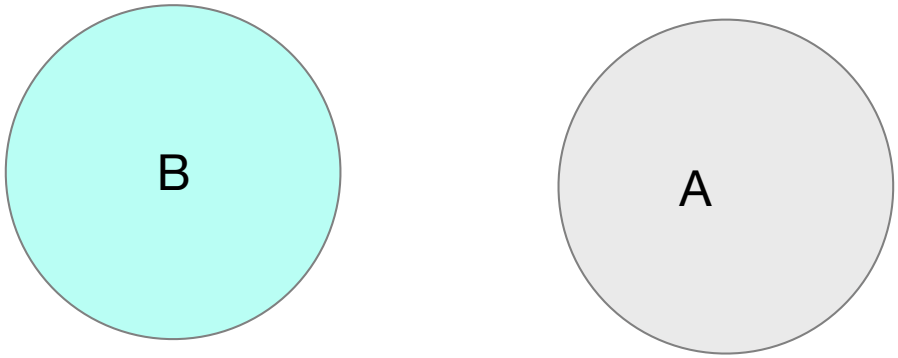
- P(A or B) = P(A) + P(B)- P(A and B)

Some rules



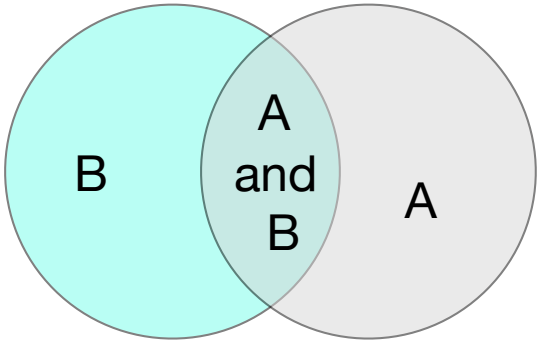
- P(A or B) = ?

Some rules



- P(A or B) = P(A) + P(B)

Some rules



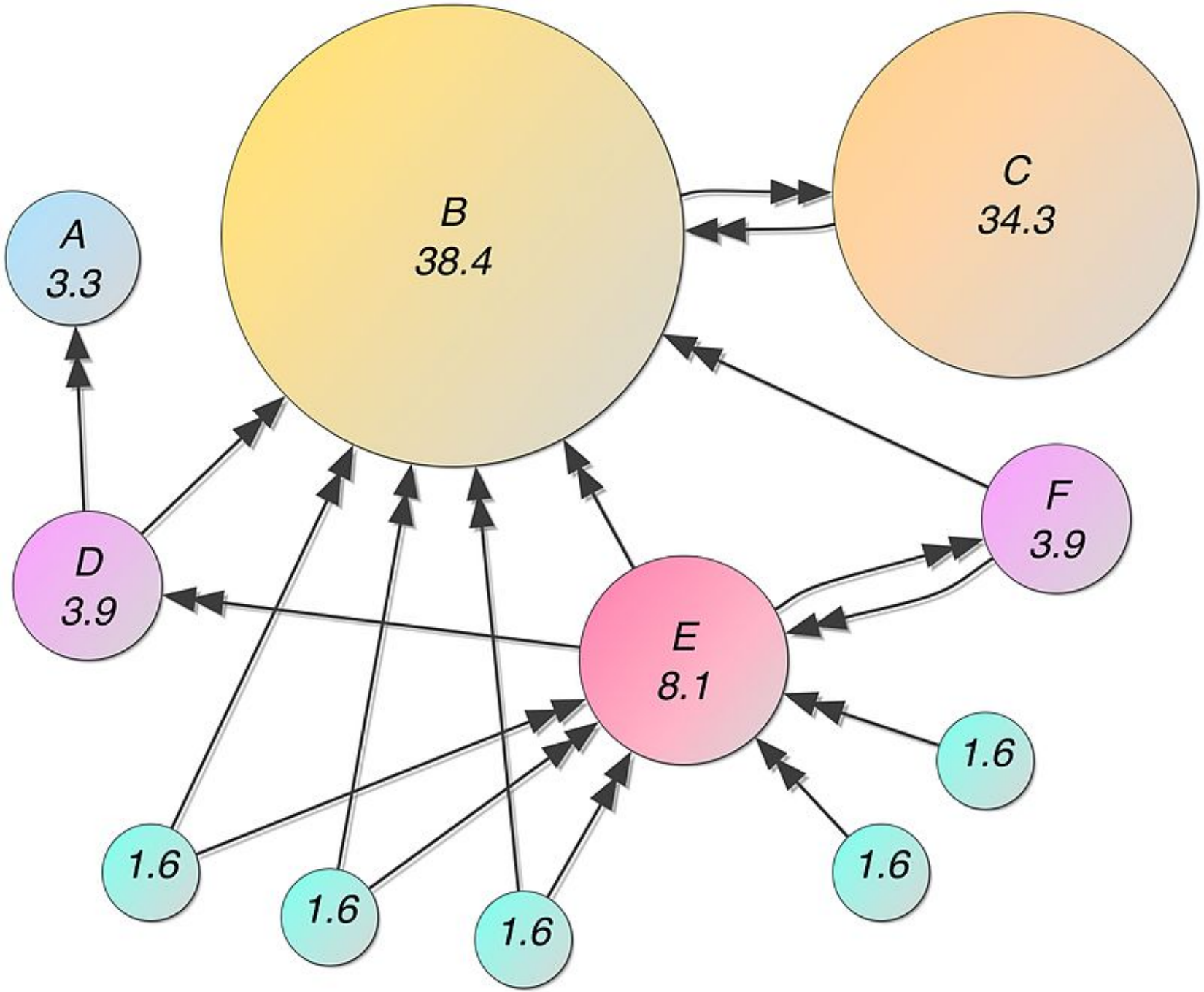
- If P(A and B) = P(A) X P(B), then A and B are called mutually independent

Applications

- Where is probability applied?
- Probability and statistics are applied everywhere — to model interesting phenomena in biology, genetics, engineering, computer science, operations research, physics, even in the social sciences
- Let's consider two fun applications …

1. PageRank Algorithm

- Developed by Larry Page and Sergey Brin - Stanford University 1996
- Founders of Google Inc.
- Main idea behind Google search


A
3.3
B
38.4
C
34.3
D
3.9
E
8.1
F
3.9
1.6
1.6
1.6
1.6
1.6

What is what?

How does Google measure the importance of a webpage?

- Importance of a page = (Page Rank of a page)/C(page A)

Main Idea

- The PageRank algorithm is based on what is known as a Markov chain
- PageRank (importance) of a webpage increases if more webpages direct to it

Andrey Markov (1856-1922)

2. Text Prediction

- Frequently used phrases have a higher probability
- So it is reasonable to assume that the next word only depends on what the previous word was
- Train your algorithm on a large amount of text:
  - I like Photography.
  - I like Science.
  - I love Mathematics.

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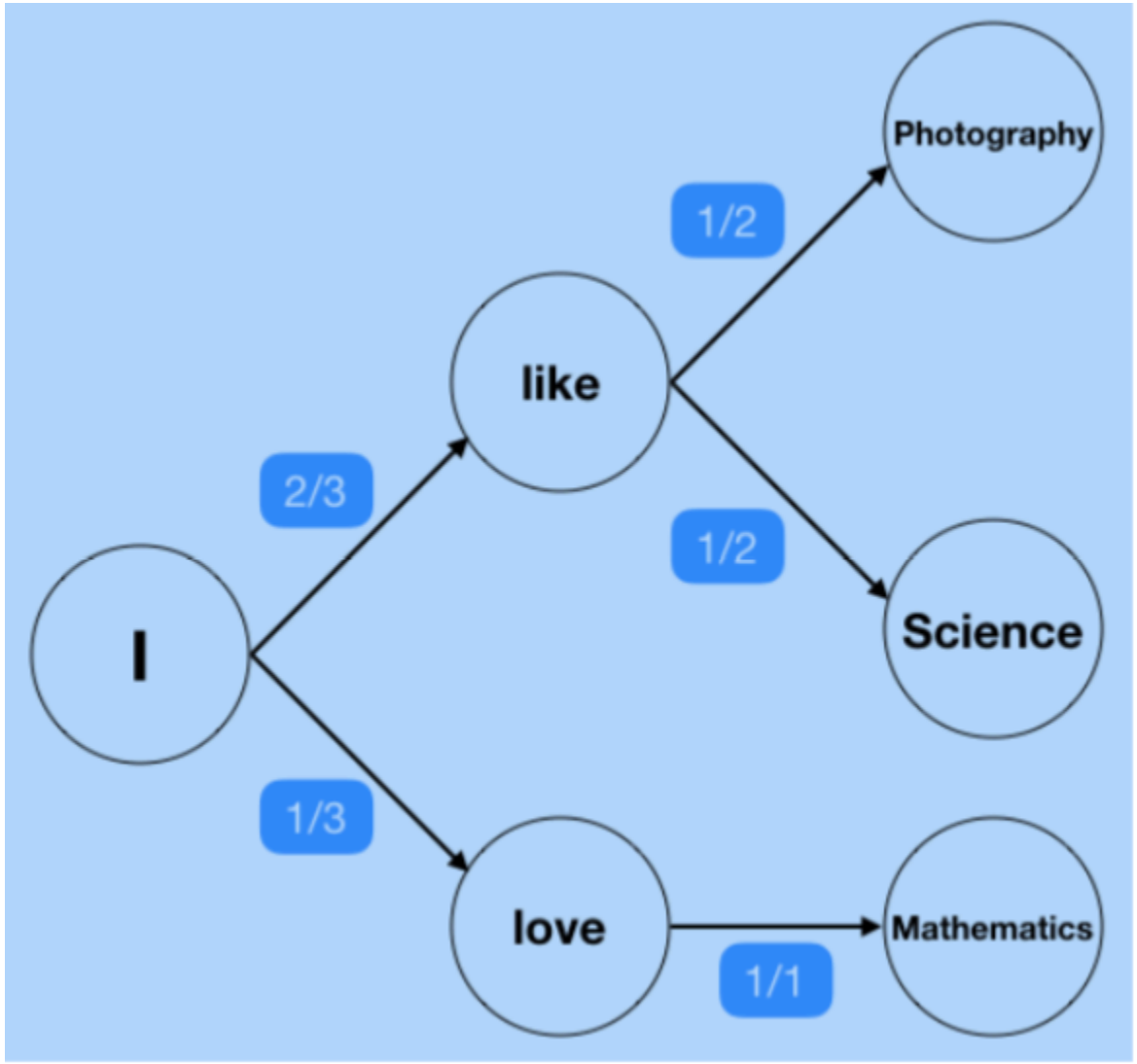

State transition diagram for our sample data