Fractals: properties and applications

MATH CO-OP

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Fractal ball experiment: DIY!
How do we think of dimension?
Conclusions: Fractal properties

• Fractals exhibit **fractal dimensions**: all objects whose dimension is not an integer are fractals.

• Fractals are **self-similar**.

\[ d = 1.2683 \]
Fractals - mathematical objects

Mandelbrot set

Variation of a Mandelbrot set
Fractals - around us

Lake Mead coastline

The Great Wave off Kanagawa - Hokusai
1. Fractal antennas

Sierpinski triangle

Example of fractal antenna
• Fractal-shape antennas can respond to more frequencies than regular ones.
• They can be ¼ the size of the regular ones: use in cellular phones and military communication hardware.
• BUT: Not all fractal shapes are best suited for antennas.
2. Coastlines

Border length

- Portugal - Spain border

987 km (reported by the Portuguese)

1214 km (reported by the Spanish)

Measurements were using different scales!
Returning to coastlines…

South Africa

Britain
Approximating a smooth curve using straight lines – guaranteed to get closer to the true value of the curve length

Can we say the same for the UK coastline?
Scale/ruler length: \( l = \frac{1}{r} \)

Perimeter/length: \( N \times l \)

\[
9 \times \frac{1}{1} = 9 \\
19 \times \frac{1}{2} = 9.5 \\
48 \times \frac{1}{4} = 12 \\
97 \times \frac{1}{8} = 12.125
\]
• Coastlines have fractal-like properties: complexity changes with measurement scale

• A lot like the Koch curve

• This curve has **infinite length**!

• **Length**: makes little sense
But, concept of **fractal dimension** makes sense!

South Africa: $d = 1.02$

Britain: $d = 1.25$
• This is called the “Coastline paradox”: measured length of a stretch of coastline depends on the measurement scale.

• But for practical use, the ruler scale is not that fine: km’s are enough!

• Approximating the coastline with an infinite fractal is thus not so useful in this case.