FERMI QUESTIONS AND ORDERS OF MAGNITUDE

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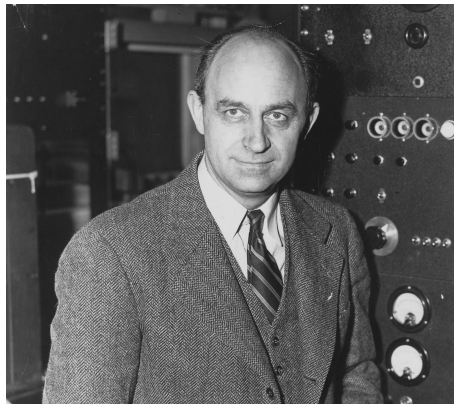
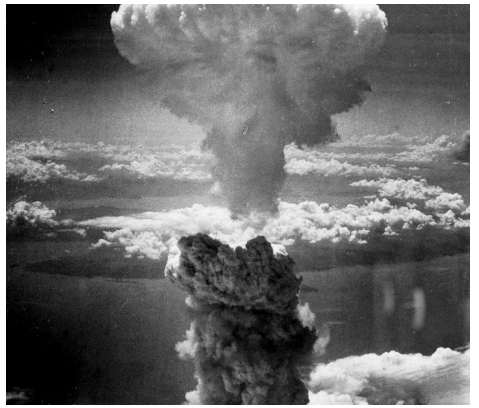
ENRICO FERMI

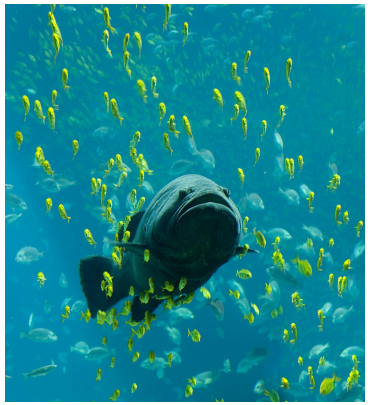
- Prominent physicist who worked on Manhattan Project during WWII
- Known for making good, fast approximations
- Estimated strength from first nuclear bomb test to within a factor of 2

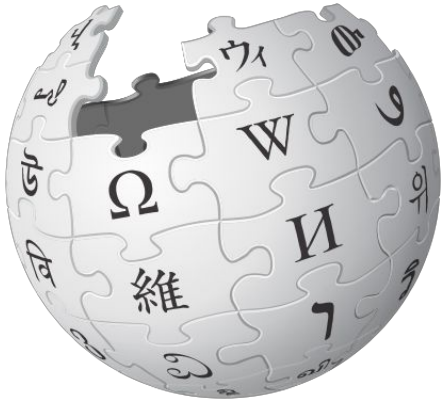




FERMI QUESTIONS

- Fermi questions are estimation problems where we have to guess some really large quantity using very little information. Things like:
  - How many fish are in the sea?
    - About 3 trillion
  - How long would it take to read all of Wikipedia?
    - About 20 years if it stopped updating
  - How many leaves are on an average, fully grown oak tree
    - About 200,000





WHAT IS A GOOD GUESS?

How close to 200,000 leaves should our guess be?

- 1 leaf is definitely too small
- 100,000,000,000,000,000 (number of leaves in the world) definitely too big
- Is "off by 5 leaves" good enough?
  - 199,995 or 200,005 both much more accurate than we could ever hope for
- Is "off by 10%" good enough?
  - 180,000 or 220,000 probably still within error of approximation
- What about 20,000? 2,000,000?

MAIN MATH TOOL: POWERS OF 10

- Sometimes, we only care about **"orders of magnitude"**
  - **Number of times you have to multiply or divide your answer by 10 to get to the right answer**
  - 20,000 is 1 order of magnitude too low, 2,000,000 is 1 order of magnitude too high
  - What about 2,000?
    - 2 orders of magnitude too low, since you have to multiply by 10 twice to get to 200,000
  - What about 50,000?
    - Less than 1 order of magnitude away, since 50,000 is too small, but 50,000x10=500,000 is already too big
- **Answer is "good enough" if it's within 1 or 2 orders of magnitude**
- Lets us simplify calculations a lot by only looking at powers of 10

Decreasing by 1 order of magnitude:

200000

↓

200000

↓

20000

Increasing by 1 order of magnitude:

200000

↓

2000000

↓

2000000

EXAMPLE

How many pennies need to be stacked to reach the height of the Empire State Building?

- How tall is the Empire State Building?
  - 1454 feet (about 10^3)
- How many inches are in a foot?
  - 12 (about 10^1)
- How many pennies in an inch-tall stack?
  - 17 (about 10^1)
- Total number of pennies needed: about $10^3 \times 10^1 \times 10^1 = 100{,}000$ (actual answer: around 280,000)

10- MINUTE GROUP ACTIVITY

How many piano tuners are there in Chicago?

WHAT HAPPENED?

- Any of the approaches from the 3 groups could have been used to solve the problem
- But some groups' answers were easier to estimate, giving them more accurate results
- Problem often comes down to **choosing the right information** to use and how to use it

10-MINUTE GROUP ACTIVITY 2

How many Oreo cookies would it take to fill this classroom?

WHY DO WE CARE ABOUT THIS?

- Techniques from Fermi questions very similar to how we solve problems in real life:
  - Open-ended problem where **we have to figure out what information we need** and how to use it
  - Can **make assumptions** about information that isn't given in the problem
  - Use the tools we have to get the best answer we can when **"exact" answer may not be possible**

THANK YOU!

- Images: Wikimedia Commons
- https://upload.wikimedia.org/wikipedia/commons/d/d4/Enrico_Fermi_1943-49.jpg
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