The Forms of Swarms: A Primer of Swarm Equilibria

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Abstract:
Biological aggregations (swarms) exhibit morphologies governed by social interactions and responses to environment. Starting from a particle model we derive a nonlocal PDE, known as the aggregation equation, which describes evolving population density. The solutions to the aggregation equation can exhibit a variety of behaviors including spreading without bound, concentrating into δ-functions and formation of compactly supported equilibria. We describe some tools for investigating the asymptotic behavior of solutions. We also study equilibria and their stability via the calculus of variations which yields analytical solutions. Finally we present a case study about how these methods can be used to construct a model of locust swarms.