Lattice paths, linear algebra and combinatorics
Day 4

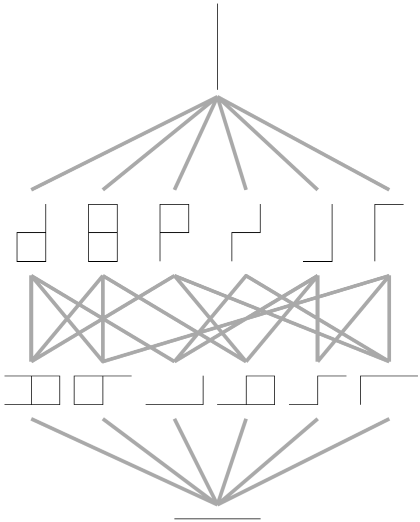
Carolina Benedetti Velásquez

Universidad de los Andes

with Jerónimo Valencia

Mathematics Sin Fronteras
November 18th, 2021

Quotients of LPMs on [3]



A summary

$$\begin{array}{c} \begin{matrix}1 & 2 & 3 & 4\end{matrix} \\ \begin{bmatrix}1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 1\end{bmatrix} \end{array}$$

Linear Independence / Bases

$\mathcal{B} = \{12, 13, 14, 24, 34\}$

$\xrightarrow{\text{Axiomatization}}$

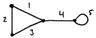
$\mathcal{M} = ([n], \mathcal{B})$ matroid iff

1. $\mathcal{B} \neq \emptyset$
2. $\forall A, B \in \mathcal{B}$:

   $\forall a \in A \setminus B \ \exists b \in B \setminus A$ s.t.

   $A \setminus \{a\} \cup \{b\} \in \mathcal{B}$.

$\xrightarrow{\text{Graphs}}$

G: 

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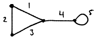

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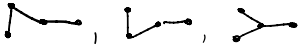
$\mathcal{B} = \{$  $\}$

Other examples: greedy algorithm; algebraic extensions; projective geometry; polytopes; etc.

Another summary

Algebra

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\uparrow$$

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Axiomatization

$$\rightsquigarrow$$

"M is a quotient of N"

iff

$\forall\, C \in \mathcal{C}(N)$:

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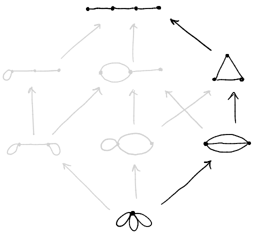
Matroids



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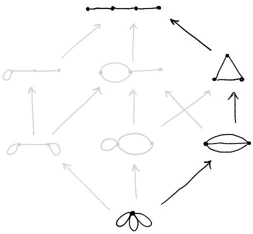
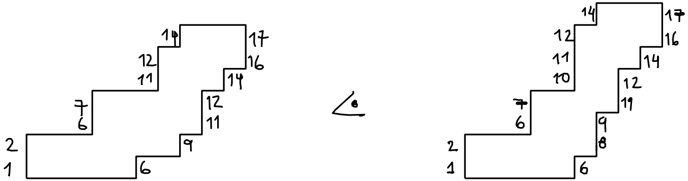


¿How does this property look in other families?

An example on [17]



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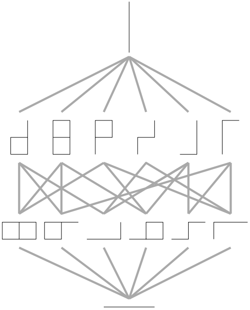

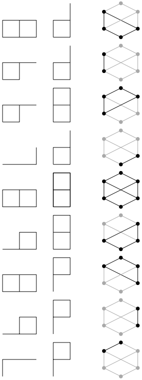

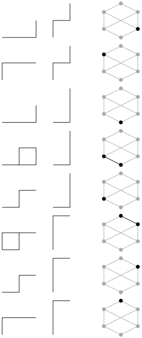

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Matroids are connected with many branches of mathematics, both pure and applied.

- Are there other axiomatizations?
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¡Muchas gracias!