Lattice paths, linear algebra and combinatorics
Day 2

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Mathematics Sin Fronteras
October 28th, 2021

La vez pasada

A matroid $M = (E, \mathcal{B})$ on the ground set E is such that $\mathcal{B}$ consists of subsets of E where:

- $\mathcal{B} \neq \emptyset$
- For all $A, B \in \mathcal{B}$, for all $a \in A - B$ there is $b \in B - A$ such that $A - a + b \in \mathcal{B}$.

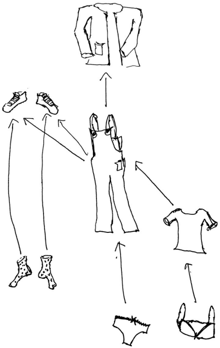
Ejemplos:

grafos: $G = (E, \mathcal{B})$ where E are the set of edges and $\mathcal{B}$ is the collection of trees.

k-ev: Let V be a k-dim. vector space in $\mathbb{R}^n$. Then $M_V = ([n], \mathcal{B})$ where $\mathcal{B} = \{J \,:\, |J| = k, p_J \neq 0\}$.

Today: Partially Ordered SETs - posets

credit: Viviane Pons



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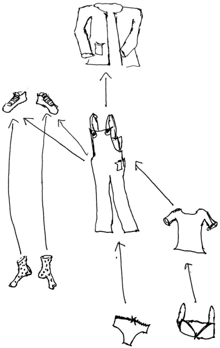

IDEA: Given a set of elements, order them according a given rule.

Posets

A partially ordered set is a pair $(P, \leq)$ where P is a set and $\leq$ is such that for all $x, y, z \in P$:

- $x \leq x$ (reflexive)
- If $x \leq y$ and $y \leq x$ then $x = y$ (antisymmetric)
- If $x \leq y$ and $y \leq z$ then $x \leq z$ (transitive).

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Ejemplos:

- $(\mathbb{N}, \leq)$.
- $(D_n, \leq)$ where $D_n = \{m \geq 1 : m|n\}$ and $k \leq m \leftrightarrow k|m$.

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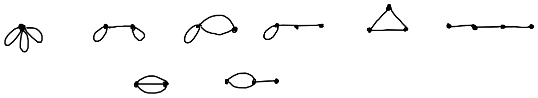
Notation: $x \lessdot y \leftrightarrow x < y$ and no z satisfies $x < z < y$ (y covers x).

Let's order graphical matroids on $[n]$



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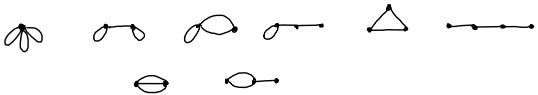
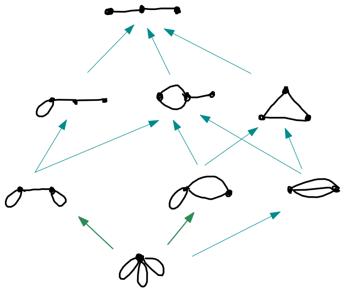
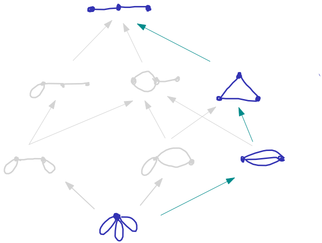




- $\circ$ $g \lessdot h \leftrightarrow$ every circuit of h is union of circuits in g.

chain of the poset

- How can we order the set of LPMs on $[n]$.
- What for?



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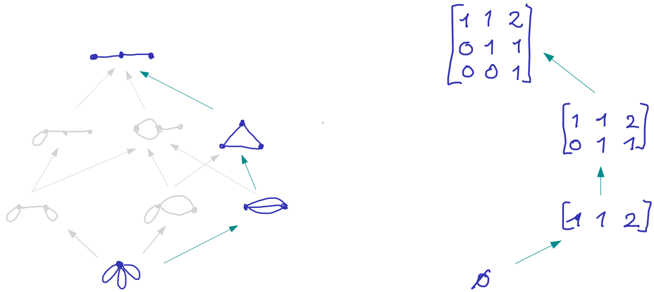
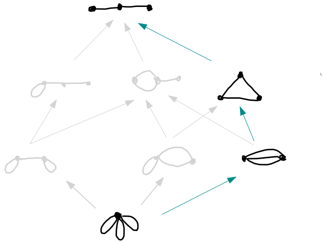


- We need a good axiomatization that captures what's happening on the RHS.

Quotients of matroids

A matroid N is a quotient of a matroid M if every circuit of M is union of circuits of N.



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- What to do in the case of LPMs? Using their combinatorics?

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¡Muchas gracias!