Homework 2
Re-Imaging the World through Linear Algebra
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Name ________________________________

1. Determine which of the following matrices are symmetric.

$$A = \begin{pmatrix} 0 & 8 & 3 \\ 8 & 0 & -2 \\ 3 & -2 & 0 \end{pmatrix}, \; B = \begin{pmatrix} -6 & 2 & 0 \\ 0 & -6 & 2 \\ 0 & 0 & -6 \end{pmatrix}, \; C = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \end{pmatrix}$$

2. Since vectors in $\mathbb{R}^n$ may be regarded as n x 1 matrices, the properties of transposes apply to vectors too. Let $A = \begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix}$ and $\mathbf{x} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$. Compute $(A\mathbf{x})^T$, $\mathbf{x}^T A^T$, $\mathbf{x}\mathbf{x}^T$, and $\mathbf{x}^T\mathbf{x}$. Is $A^T\mathbf{x}^T$ defined?

3. Compute the product AB in two ways: (a) by the definition, where $A\mathbf{b}_1$ and $A\mathbf{b}_2$ are computed separately, and (b) by the row-column rule for computing AB.

$$A = \begin{pmatrix} 4 & -3 \\ -3 & 5 \\ 0 & 1 \end{pmatrix}, \; B = \begin{pmatrix} 1 & 4 \\ 3 & -2 \end{pmatrix}$$

4. Is the sum of two Toeplitz matrices Toeplitz? What about the product? Prove your answer. The following is an example of a Toeplitz matrix:

$$T = \begin{pmatrix} 5 & 7 & 9 & 11 \\ 8 & 5 & 7 & 9 \\ 4 & 8 & 5 & 7 \\ 6 & 4 & 8 & 5 \end{pmatrix}$$

5. Is the sum of two circulant matrices circulant? What about the product? Prove your answer. The following is an example of a circulant matrix:

$$C = \begin{pmatrix} 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

6. What is the cost of computing this matrix?

$$2\begin{pmatrix} 2 & 3 & 2 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{pmatrix} + 3\begin{pmatrix} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 7 & 10 \\ 5 & 7 & 12 \\ 5 & 9 & 14 \end{pmatrix}$$

7. Check that the following equalities hold.

$$\begin{pmatrix} 5 & 5 \\ 3 & 5 \end{pmatrix} = 5\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + 5\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + 3\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} + 5\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 5 \\ 3 & 5 \end{pmatrix} = 3\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + 0\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} + 2\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} + 0\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$