Random Graphs and Social Networks: Homework 2

Mathematics Sin Fronteras, 2021

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Exercise 1:

Suppose X is a Poisson random variable with mean $\lambda > 0$, i.e.,

$$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}, \qquad k = 0, 1, 2, \ldots$$

Show that for any constant $\gamma > 0$,

$$\lim_{k\to\infty} k^\gamma P(X = k) = 0.$$

Hint: Approximate $k!$ using Stirling's formula.

Exercise 2:

Suppose we want to generate a graph with 1000 vertices and two communities. The first community, call it 1, should have 600 vertices, while the second one, call it 2, should have 400. The expected degree for vertices of community 1 should be (approximately) 5, and the expected degree of vertices in community 2 should be (approximately) 6. Give a kernel κ that would accomplish this for a stochastic block model, while simultaneously separating the two communities.