Random Graphs and Social Networks: Homework 1

Mathematics Sin Fronteras, 2021

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Exercise 1:

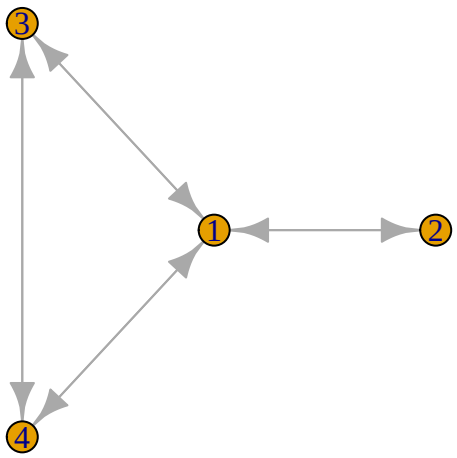
In class, we talked about different types of graphs. In this exercise, we consider only undirected graphs. For each of the following described properties, find a graph that satisfies them.

1. A simple, connected graph with 6 vertices, and no cycles.
2. A simple, complete graph on 5 vertices.
3. A simple graph on 10 vertices, with 2 connected components.
4. A connected multigraph with 4 vertices and 6 edges.

For the graphs (1) and (2) and (3) that you drew, determine the diameter, and write the degree of each vertex besides them.

Exercise 2:

In class, it was stated that the k-th power of the adjacency matrix A^k contains the number of paths of length k going from any two vertices. In this exercise, we explore this interesting relationship. Consider the following undirected graph on 4 vertices:



1. Find the adjacency matrix A associated with the given graph.
2. Find the matrices A^2, A^3 and A^4.
3. The entry $(1, 4)$ of A^4 is 6. This indicates there are exactly 6 paths starting from 1 and ending on 4 of length 4. Draw these six paths.

Exercise 3:

Often, when a directed graph has a strongly connected component some vertices may lie outside of this connected component. The vertices that can be reached by the component, but cannot themselves reach the component are known as the "fan-out." The vertices that can reach the component but cannot be reached back are known as "fan-in." A large strongly connected component with non-empty fan-in and fan-out are usually called "bow-tie" components. Draw a directed graph on 12 vertices with a strongly connected component of size 5, a fan-in of size 3 and a fan-out of size 4.