1. \textit{(graphical matroids)}

Let $M$ be the graphical matroid arising from the graph depicted below.

(a) Provide an example of two bases of $M$ and verify the exchange axiom for the chosen bases.
(b) Give examples of independent, dependent, circuits, elements of $M$.
(c) Prove that $M$ is indeed a matroid (i.e. the basis exchange axiom is satisfied).
(d) Classify all the graphical matroids on the ground set $[3]$.

2. \textit{(LPMs)}

(a) Give an example of an LPM on $[12]$ of rank 6.
(b) Choose two bases of the matroid $M$ provided in (a). Verify the exchange axiom.
(c) Prove that the exchange axiom works for LPMs.

3. \textit{(Independent set axioms)} A matroid $M = (E, \mathcal{I})$ is such that the collection $\mathcal{I}$ satisfies:

\begin{itemize}
    \item \textbf{(I0)} $\emptyset \in \mathcal{I}$
    \item \textbf{(I1)} If $I \in \mathcal{I}$ and $K \subseteq I$ then $K \in \mathcal{I}$.
    \item \textbf{(I2)} If $I, J \in \mathcal{I}$ are such that $|I| < |J|$ then there is $j \in J - I$ such that $I \cup \{j\} \in \mathcal{I}$.
\end{itemize}

(a) Verify that the linear matroid $M_{V}$ seen in class satisfies these axioms.
(b) Prove that any linear matroid satisfies these axioms.