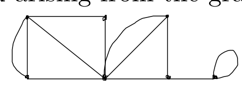
# MSF - TAREA 1

CAROLINA BENEDETTI VELÁSQUEZ AND JERÓNIMO VALENCIA

1. (*graphical matroids*)
   Let $M$ be the graphical matroid arising from the graph depicted below.



   (a) Provide an example of two bases of $M$ and verify the exchange axiom for the chosen bases.
   (b) Give examples of independent, dependent, circuits, elements of $M$.
   (c) Prove that $M$ is indeed a matroid (i.e. tha basis exchange axiom is satisfied).
   (d) Classify all the graphical matroids on the ground set [3].

2. (*LPMs*)
   (a) Give an example of an LPM on [12] of rank 6.
   (b) Choose two bases of the matroid $M$ provided in (a). Verify the exchange axiom.
   (c) Prove that the exchange axiom works for LPMs.

3. (*Independent set axioms*) A matroid $M = (E, \mathcal{I})$ is such that the collection $\mathcal{I}$ satisfies:
   ($I0$) $\emptyset \in \mathcal{I}$
   ($I1$) If $I \in \mathcal{I}$ and $K \subseteq I$ then $K \in \mathcal{I}$.
   ($I2$) If $I, J \in \mathcal{I}$ are such that $|I| < |J|$ then there is $j \in J - I$ such that $I \cup \{j\} \in \mathcal{I}$.
   (a) Verify that the linear matroid $M_V$ seen in class satisfies these axioms.
   (b) Prove that any linear matroid satisfies these axioms.