An introduction to error correcting codes

Exercises II

Exercise 1. Prove that if $\mathcal{C}$ is a code over $\mathbb{F}_q$ then $\mathcal{C}^\perp$ is also a code over $\mathbb{F}_q$. In other words, prove that $\mathcal{C}^\perp$ is vector subspace of $\mathbb{F}_q^n$.

Exercise 2. Prove that if $\mathcal{C}$ is a code then $(\mathcal{C}^\perp)^\perp = \mathcal{C}$.

Exercise 3. Prove that if H is parity check matrix for $\mathcal{C}$ then we have

$$x \in \mathcal{C} \iff Hx^T = 0.$$

Exercise 4. Construct a table of coset leaders and syndromes for the binary code with generator matrix

$$G = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}.$$

Write the parameters of this code. How many errors can this code correct? How many can detect?

Exercise 5. Consider the binary code $\mathcal{C}$, i.e. $\mathbb{F}_2$, with length 6 and dimension 3 generated by the matrix

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

We saw that $\mathcal{C} = \{(000000), (100011), (010101), (001110), (110110), (011011), (101101), (111000)\}$ and that

$$H = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

is a parity check matrix for $\mathcal{C}$. Use the following table with coset leaders and syndromes

$e_0 = (000000) \rightarrow S(e_0) = (000)$	$e_4 = (000100) \rightarrow S(e_4) = (100)$
$e_1 = (100000) \rightarrow S(e_1) = (011)$	$e_5 = (000010) \rightarrow S(e_5) = (010)$
$e_2 = (010000) \rightarrow S(e_2) = (101)$	$e_6 = (000001) \rightarrow S(e_6) = (001)$
$e_3 = (001000) \rightarrow S(e_3) = (110)$	$e_7 = (100100) \rightarrow S(e_7) = (111)$

to correct errors made in the transmission if the received word is:

- $y = (010001)$
- $y = (011101)$
- $y = (011111)$
- $y = (110010)$
- $y = (110101)$