An introduction to error correcting codes

Exercises

Exercise 1. Prove that the Hamming distance d is a metric in $\mathbb{F}_q^n$. In other words, prove that:

(i) $d(x, y) = 0$ if and only if $x = y$ and $d(x, y) \geq 0$ for all $x, y \in \mathbb{F}_q^n$;

(ii) $d(x, y) = d(y, x)$ for all $x, y \in \mathbb{F}_q^n$;

(iii) $d(x, z) \leq d(x, y) + d(y, z)$ for all $x, y, z \in \mathbb{F}_q^n$.

Recall that $d : \mathbb{F}_q^n \times \mathbb{F}_q^n \longrightarrow [0, n]$ is defined by $d(x, y) = \#\{i : x_i \neq y_i, 1 \leq i \leq n\}$ where $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$.

Exercise 2. Prove that if $x, y \in \mathbb{F}_q^n$ then $d(x, y) = w(x - y)$, where w represents the weight defined by $w(z) = \#\{i : z_i \neq 0, 1 \leq i \leq n\}$. Use this to prove that for any linear code $\mathbb{C}$ the minimum distance is equal to the minimum weight.

Exercise 3. Consider the binary code C_1, i.e. $\mathbb{F}_2$, with length 4 and dimension 2 generated by the matrix

$$G = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}.$$

We saw that $C_1 = \{(0000), (1011), (0101), (1110)\}$ and that the minimum distance of this code $d(C_1) = 2$.

Write the generator matrix of the code $C_2 = \{(0000), (0011), (1100), (1111)\}$. Which are the parameters of C_2? Is C_2 equivalent to C_1?

Recall that two codes are said to be equivalent if one can be obtained by applying a fixed permutation of the positions to all the codewords of the other.