Homework 1
Re-Imaging the World through Linear Algebra
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Name ______________________________

1. Use the vectors $\mathbf{u} = (u_1, \ldots, u_n)$, $\mathbf{v} = (v_1, \ldots, v_n)$, and $\mathbf{w} = (w_1, \ldots, w_n)$ to verify the following algebraic properties of $\mathbb{R}^n$.

   a. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$

   b. $k(\mathbf{u} + \mathbf{v}) = k\mathbf{u} + k\mathbf{v}$ for each scalar k.

2. Use the vector $\mathbf{u} = (u_1, \ldots, u_n)$ to verify the following algebraic properties of $\mathbb{R}^n$.

   a. $\mathbf{u} + (-\mathbf{u}) = (-\mathbf{u}) + \mathbf{u} = \mathbf{0}$

   b. $k(k'\mathbf{u}) = (kk')\mathbf{u}$ for all scalars k and k'.

3. Consider the vectors $\mathbf{u} = (2, -7, 1)$, $\mathbf{v} = (-3, 0, 4)$, and $\mathbf{w} = (0, 5, -8)$. Compute a.) $3\mathbf{u} - 4\mathbf{v}$ and b.) $2\mathbf{u} + 3\mathbf{v} - 5\mathbf{w}$.

4. Verify that the vectors $\mathbf{u} = (-1, 0, 1)$, $\mathbf{v} = (2, 4, 2)$, and $\mathbf{w} = (3, -3, 3)$ are orthogonal to each other.

5. Compute the Euclidean norm of the vectors $\mathbf{u} = (-1, 0, 1)$ and $\mathbf{v} = (2, 4, 2)$ and the distance between them.

6. The 1-norm of a real vector is defined by $\|\mathbf{v}\|_1 = \sum_{i=1}^{n} |v_i|$. Compute the 1-norm of the vectors $\mathbf{u} = (1, 2, 3)$ and $\mathbf{v} = (3, 2, 1)$.

7. The ∞-norm of a real vector is defined by $\|\mathbf{v}\|_\infty = \max_i |v_i|$. Compute the ∞-norm of the vectors $\mathbf{u} = (1, 2, 3)$ and $\mathbf{v} = (3, 2, 1)$.

8. Decide which of the following statements are TRUE and which are FALSE. For the TRUE ones, you have to give a proof. For the FALSE ones, you have to give a counterexample. Here we are considering $\mathbf{u} = (u_1, \ldots, u_n)$, $\mathbf{v} = (v_1, \ldots, v_n)$, and $[\mathbf{u}, \mathbf{v}] = (u_1, \ldots, u_n, v_1, \ldots, v_n)$ is the vector formed by concatenating them.

   a. $\|\mathbf{u}\|_1 + \|\mathbf{v}\|_1 = \|[\mathbf{u}, \mathbf{v}]\|_1$

   b. $\|\mathbf{u}\|_2^2 + \|\mathbf{v}\|_2^2 = \|[\mathbf{u}, \mathbf{v}]\|_2^2$

   c. $\|\mathbf{u}\|_\infty + \|\mathbf{v}\|_\infty = \|[\mathbf{u}, \mathbf{v}]\|_\infty$

9. Compute the following:
$$4\begin{pmatrix} 1 & 2 \\ 4 & 6 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix} - 2\begin{pmatrix} 5 & 1 \\ 4 & 3 \end{pmatrix}.$$

10. Find the values of x, y, z, and w so that the following matrix equality holds:
$$3\begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} x & 6 \\ -1 & 2w \end{pmatrix} + \begin{pmatrix} 4 & x+y \\ z+w & 3 \end{pmatrix}.$$