Treatment of Nonlinear Stochastic PDE with Fractional Derivative Terms: Iterative Analytical Solution and Monte Carlo Simulations

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Abstract

The paper deals with a nonlinear stochastic partial differential equation involving fractional derivative terms and stochastic processes as forcing functions. The mathematical model encapsulates the physical modelling of nonlinear random vibrations of a beam; the nonlinear term arises from the assumption of moderately large system displacements. It is shown that the solution of the PDE can be reliably determined via an optimal statistical linearization procedure. Specifically, the solution is obtained by utilizing an appropriate iterative representation of the stochastic response spectrum, which involves the linear modes (eigenfunctions) of vibration of the beam. Such a representation allows retaining the nonlinearity in the time dependent part of the response, which, in turn, is linearized in the time domain in a stochastic mean square sense.

The reliability of the proposed approximate solution is assessed vis-a-vis the results of relevant Monte Carlo simulations. In this regard, a recently proposed BIEM (Boundary Integral Equation Method) based algorithm is fully exploited for estimating the PDE solution from spectrum compatible realizations of the excitation. The algorithm has the quite desirable feature of readily accounting for the memory effect due to the fractional derivative. The presented analytical and Monte Carlo results pertain to simply supported beams excited by a random uniform load of a given power spectral density function (coloured white noise).