Numerical solution of the space-time fractional diffusion equation: Alternatives to finite differences.

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One of the ongoing issues with fractional-order diffusion models is the design of efficient numerical schemes for the space and time discretizations. Until now, most models have relied on a low-order finite difference (FD) method to discretize both the fractional-order space and time derivatives. Some numerical schemes using low-order finite elements (FE) have also been proposed. Both the FD and FE methods have long been used to solve integer-order partial differential equations. These low-order schemes generally require many grid points to obtain an accurate solution but that is offset by their local or piecewise nature. They lead to systems of linear equation defined by large sparse matrices that can be handled easily. However, fractional-order derivatives are non-local differential operators and the resulting matrices are full as the global behavior of the solution has to be taken into account. The computational efficiency of FD and FE schemes is thus severely impaired when going from an integer-order to a fractional-order equation.

In this talk, we consider high-order, global numerical methods like the pseudo-spectral (PS) and radial basis functions (RBF) methods to discretize the space and time fractional derivatives. These methods appear to be a better choice as they naturally take the global behavior of the solution into account and use a limited number of degrees of freedom. Our PS scheme relies on an expansion of the model solution in terms of Chebyshev polynomials in both space and time. Our RBF scheme relies on the QR algorithm to analytically remove the ill-conditioning that appears when the number of nodes increases or when basis functions are made increasingly flat.

We will present a flexible approach that allows the combination of a Chebyshev PS expansion in time with either FD, FE, PS or RBF discretizations in space. The proposed method can also accommodate an exponential tempering of the space and/or time fractional derivatives. A number of examples of numerical solutions of the space-time fractional diffusion equation are presented with various combinations of the time and space derivatives. The proposed numerical scheme is shown to be both efficient and flexible.

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