Recent advances on numerical solution of fractional PDEs

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This report surveys recent advances on numerical solution of fractional PDEs, which describe "anomalous" mechanical and physical behaviors of contaminant transport and power law acoustic dissipation in porous media. The term "anomalous" is considered to represent that, in the above-mentioned cases, various standard gradient laws of physics and mechanics, e.g., Fickian diffusion, Fourier heat conduction, Darcy's law, are broken, and the fractional power law appears. The standard mathematical models of integer-order derivatives can not accurately reflect such behaviors, while the fractional derivatives are instead found in recent decades a powerful mathematical tool for modeling such complex phenomena.

In the last decade, growing attention has been attracted to numerical solution of the fractional PDEs. However, it is far from being mature for practical applications due to the long-range correlation (non-local memory) effect arising from the convolution integral expressions of time (space) fractional derivative. The traditional numerical techniques such as finite element, finite difference, finite volume methods and step-by-step time integration schemes, which are originally designed for integer-order partial differential equations, encounter an exponential increase of computing costs with advancing time and large spatial domain. Therefore, the innovative numerical techniques are highly demanded for efficient and stable solution of fractional PDEs.

This report will survey the status of numerical fractional PDEs and then present a variable-order fractional diffusion model to depict the decelerating/accelerating diffusion behaviors, and introduce a recent semi-analytical method for time fractional PDEs to significantly mitigate the burden of long time-history fractional diffusion systems, and give some emerging approximation strategies for fractional space derivative equations in medical ultrasonic imaging. In the conclusions, we will point out the emerging techniques to tackle the bottleneck problems in the numerical solution of fractional PDEs:

(1) Fast dense matrix algorithms to further reduce memory requirements and CPU time for large spatial domain problems;

(2) Meshless methods for fractional space derivative equations with complex-shaped boundary;

(3) Development of the software package for fractional PDEs.

References:

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