A Spectral Element Based Parareal Method for the Long Time Integration of Time-Fractional Differential Equations

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Abstract

It remains a challenging task to design efficient and accurate temporal discretization schemes for the long time integration of fractional-in-time differential equation. To ensure accuracy over long time, high-order accurate schemes are needed, yet one must be cognizant of the high cost of simple methods and algorithms. To address this, we consider the parallel-in-time (parareal) method, viewed as a predictor-corrector or multi shooting, global algorithm across the whole time domain as that appears to be well matched to the global nature of the fractional-in-time problem.

However, the combination of the parareal method and the need for high-order accuracy introduces a number of challenges associated with the global iterative procedure and the accuracy of the evaluation of the singular integral. In this paper, we introduce a new spectral element framework for time-fractional differential equations, utilizing the parareal approach to reduce the computational cost by taking advantage of parallelism. The computational time-line is decomposed into elements in each of which a spectral method is used to solve for the numerical solution, element by element. To accurately integrate the memory part of the fractional-in-time operators, with a singular or nearly singular kernel, two essential techniques are introduced: the evaluation near the kernel singularity is based on the evaluation of a three-term-reccurence relation of Jacobi polynomials and the evaluation away from the kernel singularity is based on high-order Gauss quadratures. A modified parareal method is applied to solve the time-fractional problem and we shall demonstrate that our hybrid spectral element approach achieves hp-convergence for very long time integrations. The numerical results clearly demonstrate the feasibility of the parallel-in-time approach for the fractional-in-time problem and the potential for substantial accelerations on a parallel/many-core computational architecture.

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