Fractional Dynamics of a Decision Making Network

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April 7, 2013

Abstract

A two-state master equation (TSME), the decision making model (DMM), that we previously developed has been shown to generate phase transitions, to be topologically complex and to manifest temporal complexity through an inverse power-law (IPL) probability distribution function (*pdf*) in the switching times between two critical states of consensus. These properties are entailed by the fundamental assumption that the network elements in the DMM imperfectly imitate one another leading to criticality in the global variable. On the other hand, the individual only directly interacts with its nearest neighbors on a two-dimensional lattice in the TSME and is not aware of the change in phase of the network dynamics. A process of subordination is used to establish that a single network element can be described by a fractional master equation whose analytic solution yields the observed IPL *pdf* obtained by numerical integration of the TSME to a high degree of accuracy. In this way a stochastic fractional equation of motion for a single element replaces the directly calculated influence of the other 9,999 dynamic elements of the complex network on the individual of interest. The solution is shown to be remarkably different in the subcritical, critical and supercritical dynamic regions in agreement with the numerical calculations. The linear stochastic FME model of the individual's dynamics provides a faithful representation of the dynamic influence of a highly nonlinear complex dynamic network with 10^4 elements on the individual.