Fractional filtering problem and associated fractional Zakai equation

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The filtering problem has applications in many areas, including engineering, finance, and biology. The classical linear filtering problem was studied by Kalman and Bucy in the 1960th. They reduced the solution of the filtering problem to a stochastic differential equation and a deterministic Riccati type equation. In the nonlinear case the filtering problem is also connected with a stochastic differential equation, however the associated stochastic differential equation is complex and has infinite dimensions. Later Zakai succeeded to reduce the solution of the nonlinear filtering problem to a linear stochastic partial differential equation for unnormalized filtering density. This equation is called a Zakai equation.

In the talk we discuss a wide generalization of the nonlinear filtering problem, which we call a fractional filtering problem. Fractional model of the filtering problem significantly extends the scope of the filtering problems both theoretically and their engineering and other applications. The associated Zakai equation contains fractional order derivative in the sense of Riemann-Liuoville, and therefore naturally called a fractional Zakai equation. In the talk we demonstrate a derivation of the fractional Zakai equation for fractional filtering problems driven by Brownian motion and Levy processes.