# Matrix-based approaches as an emerging framework for numerical solution of initial and boundary value problems for ordinary and partial differential equations of arbitrary real order

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The variety of matrix-based techniques for numerical solution of differential equations that appeared in recent years clearly indicates that this is can be considered as a rapidly emerging general framework. Although not all of those matrix-based approaches were initially developed for fractional differential equations, some of them can be extended to such type of problems. A survey and classification of available matrix-based approaches to numerical differentiation and integration is introduced.

The so-called "matrix approach", developed by the author, to discretization of operators of arbitrary real (integer and non-integer) order and to numerical solution of fractional-order differential and integral equations is presented in detail. This method allows easy, uniform, and transparent solution of ordinary fractional differential equations, partial fractional differential equations, and ordinary fractional integral equations. It works for equations with a mixture of left-sided, right-sided, and symmetric fractional-order operators, for equations with derivatives of variable orders, distributed orders, and for equations with delayed fractional-order derivates. Recent improvements include the extension of this method to non-uniform grids and variable step lengths. The Matlab toolboxes that implement this method are presented, too, and are used in illustrative examples of numerical solution of various standard types of differential equations of non-integer orders.

In conclusion, further possibilities, directions and challenges in the field of matrix-based approaches to numerical solution of initial value problems, boundary value problems, and problems with other types of additional conditions are discussed.