“Taming fractional diffusion: bounded domains and tempered Lévy processes”

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The fractional diffusion (FD) equation is usually derived as the macroscopic limit of a continuous time random walk in an *unbounded domain* with jump distribution function given by an alpha-stable Lévy processes with *divergent second order moment*. However, transport problems of practical interest take place in bounded domains and involve observables with finite moments. Motivated by this, in this talk we discuss the formulation of fractional diffusion models in bounded domains, and fractional models based on stochastic processes with finite moments. In the case of finite domains, the main technical difficulty is that the Riemann-Liouville fractional derivative is singular at the boundaries. To circumvent this problem we discuss a well-posed, regularized fractional diffusion (RFD) models in one-dimensional and two-dimensional domains [1]. On the other hand, to address the potentially unphysical divergence of the second order moment, we reconsider the continuous time random walk model for the case of general Lévy stochastic processes for the jumps, and obtain a general integro-differential equation describing the dynamics in the fluid, continuum limit. In the particular case of exponentially tempered Lévy processes (which have finite moments) we derive a tempered fractional diffusion (TFD) equation, which describes the interplay between memory, long jumps, and tempering effects in the intermediate asymptotic regime [2]. The dynamics exhibits a transition from super-diffusive to sub-diffusive transport with a crossover time scaling algebraically with the tempering length scale. The asymptotic behavior of the propagator (Green’s function) exhibits a transition from algebraic decay at short times to stretched Gaussian decay at long times. We also study the role of tempered Lévy processes in the super-diffusive (exponentially fast) propagation of fronts in the non-local Fisher-Kolmogorov equation [3], and in the dynamics of directed (ratchet) currents in the non-local Fokker-Planck equation [4]. We discus numerical methods for the solution of the RFD and TFD equations [1,4], and discuss applications to transport modeling in fluids and plasmas [1,5].

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