DISCONTINUOUS GALERKIN METHOD (DGM) FOR FRACTIONAL DIFFUSION PROBLEMS

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Abstract. In this talk, we propose a time-stepping DGM for the numerical solution of fractional sub-diffusion problems of the form $\partial_t u - \partial_t^{-\alpha} \nabla^2 u = f$ on $(0,T) \times \Omega$ with $-1 < \alpha < 0$. We derive generic hp-version error estimates after proving the well-posedness of the approximate solution. By employing geometrically refined time-steps and linearly increasing approximation orders, we show exponential rates of convergence in the number of temporal degrees of freedom. Moreover, for h-version DG approximations on appropriate graded meshes near $t = 0$, we claim that the error is of order $O(k^{p+1+\frac{\alpha}{2}})$, where k is the maximum time-step size and $p \geq 1$ is the degree of the time-stepping DG solution.

For the spatial discretization of our model problem, we use the hybridizable DGM and show optimal algebraic spatial error estimates assuming that the exact solution is sufficiently regular. Thus, if the approximations are taken to be piecewise polynomials of degree $r \geq 0$, the approximations to u in the $L_\infty\big(0,T;L_2(\Omega)\big)$-norm and to $-\nabla u$ in the $L_\infty\big(0,T;\mathbf{L}_2(\Omega)\big)$-norm are proven to converge with the rate h^{r+1}, where h is the maximum diameter of the elements of the spatial mesh. Moreover, for $r \geq 1$ and quasi-uniform meshes, we obtain a superconvergence result which allows us to compute, in an elementwise manner, a new approximation for u converging with a rate faster than $\sqrt{\log(Th^{-2/(\alpha+1)})}\, h^{r+2}$.

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