DISCONTINUOUS GALERKIN METHODS FOR FRACTIONAL DIFFUSION PROBLEMS

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Abstract.

Part 1 We propose a time-stepping discontinuous Galerkin method for the numerical solution of fractional sub-diffusion problems of the form $\partial_t u - \partial_t^{-\alpha}\nabla^2 u = f(t)$ with $-1 < \alpha < 0$. We derive generic hp-version error estimates after proving the well-posedness of the approximate solution. By employing geometrically refined time-steps and linearly increasing approximation orders, we show exponential rates of convergence in the number of temporal degrees of freedom for solutions with singular behavior near $t = 0$ caused by the weakly singular kernel. Moreover, for h-version DG approximations on appropriate graded meshes near $t = 0$, we proved that the error is of order $O(k^{p+1+\frac{\alpha}{2}})$, where k is the maximum time-step size and $p \geq 1$ is the degree of the time-stepping discontinuous Galerkin solution.

Part 2 I this part I talk about the use of the hybridizable discontinuous Galerkin method for numerically solving our fractional diffusion problem. For exact time-marching, we derive optimal algebraic error estimates assuming that the exact solution is sufficiently regular. Thus, if for each time $t \in [0,T]$ the approximations are taken to be piecewise polynomials of degree $r \geq 0$ on the spatial domain Ω, the approximations to u in the $L_\infty\big(0,T;L_2(\Omega)\big)$-norm and to $-\nabla u$ in the $L_\infty\big(0,T;\mathbf{L}_2(\Omega)\big)$-norm are proven to converge with the rate h^{r+1}, where h is the maximum diameter of the elements of the mesh. Moreover, for $r \geq 1$ and quasi-uniform meshes, we obtain a superconvergence result which allows us to compute, in an elementwise manner, a new approximation for u converging with a rate faster than $\sqrt{\log(Th^{-2/(\alpha+1)})}\; h^{r+2}$, for quasi-uniform meshes. These results hold uniformly in α on any closed subinterval of $(-1,0)$ provided the exact solution is smooth.

I end my talk with with a series of numerical simulations.

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