Tempered Fractional Calculus

Mark M. Meerschaert
Department of Statistics and Probability
Michigan State University
East Lansing, MI

Fractional Calculus has a close relation with Probability. Random walks with heavy tails converge to stable stochastic processes, whose probability densities solve space-fractional diffusion equations. Continuous time random walks, with heavy tailed waiting times between particle jumps converge to non-Markovian stochastic limits, whose probability densities solve time-fractional diffusion equations. Time-fractional derivatives and integrals of Brownian motion produce fractional Brownian motion, a useful model in many applications. Fractional derivatives and integrals are convolutions with a power law. Including an exponential term leads to tempered fractional derivatives and integrals. Tempered stable processes are the limits of random walk models where the power law probability of long jumps is tempered by an exponential factor. These random walks converge to tempered stable stochastic process limits, whose probability densities solve tempered fractional diffusion equations. Tempered power law waiting times lead to tempered fractional time derivatives, which have proven useful in geophysics. Applying this idea to Brownian motion leads to tempered fractional Brownian motion, a new stochastic process that can exhibit semi-long range dependence. The increments of this process, called tempered fractional Gaussian noise, provide a useful new stochastic model for wind speed data.