Stochastic Models for Fractional Subdiffusion with Reactions and Forcing

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A major theoretical challenge of the past decade in anomalous diffusion research has been to derive appropriate evolution equations for the probability density function of subdiffusive transport taking into account further complications from force fields and reactions.
Starting with the continuous time random walk (CTRW) as the underlying stochastic process we derive the generalized master equation for an ensemble of particles undergoing reactions whilst being subject to an external force field. The reactions are treated as birth and death processes and the forces are incorporated as biased random walks. We first derive the master equation for a single particle undergoing a CTRW with a pure death process in a space- and time- dependent force field. We then consider an ensemble of such CTRWs, with the ensemble composed of all initial particles and subsequent particles generated by birth processes.
From the generalized master equation we show reductions to special cases; including the fractional reaction diffusion equation for subdiffusion with nonlinear reactions kinetics and the fractional Fokker-Planck equation for subdiffusion in a space- and time- dependent force field.

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