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Title of lecture:

The fundamental solution of the distributed order fractional wave equation in one space dimension is a probability density

For the spatially one-dimensional fractional wave equation Schneider and Wyss have shown that its fundamental solution is non-negative. For the time-fractional diffusion equation Chechkin et al. have shown, using an integral formula of subordination, that for diffusion even in the case of distributed orders of temporal derivative the fundamental solution is non-negative. Naturally one suspects that the fractional wave equation also allows distributed orders for this non-negativity. However, the proof is not so easy, the methods for the mentioned simpler cases cannot straightforwardly be generalized. We (the authors of [2]) have found a proof by essential use of special classes of functions, namely of *completely monotone functions, Bernstein functions, complete Bernstein functions, Strieltjes functions.* The theory of these function classes is lucidly presented in the book [5] that was a valuable guide for our investigations. It is easy to see that this fundamental solution is normalized, hence is a probability density evolving in time. There arises the question for a stochastic process modelled by it. Another question is whether in the case of the combined distributed order diffusion-wave equation, with the orders ranging over the interval (0, 2], non-trivial in both partial intervals (0, 1] and (1, 2], the fundamental solution is non-negative. For some very special cases this non-negativity indeed holds, see [4]. This lecture is fruit of the speaker's collaboration with Yuri Luchko and Mirjana Stojanovi\´c.

A few references for background literature:

[1] A.V. Chechkin, R. Gorenflo, I.M. Sokolov: Retarding sub-diffusion and accelerating super-diffusion by distributed order fractional diffusion equations. *Phys Rev. E***66** (2002), 045129/1-7.

[2] R. Gorenflo, Yu. Luchko, M. Stojanovi\´c: Fundamental solution of a distributed order time-fractional diffusion-wave equation as probability density. *Fract. Calc. Appl. Anal.* **16** (2013), 297-316. DOI: 10.2478/s13540-013-0019-6.

[3] F. Mainardi: *Fractional Calculus and Waves in Linear Viscoelasticity.* Imperial College Press, London, 2010.

[4] E. Orsingher and L. Beghin: Time-fractioinal telegraph equations and telegraph processes with Brownian time. *Prob. Theory Rel. Fields* **128** (2004), 141-160.

[5] R.L. Schilling, R. Song, Z. Vondra\`´cek: *Bernstein Functions. Theory and Applications.* De Gruyter, Berlin 2010.

[6] W.R. Schneider and W. Wyss: Fractional diffusion and wave equations. *J- Math. Physics* **30** (1989), 134-144.

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