Exponential integrators for fractional PDEs

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Exponential integrators have been proven to be a class of effective methods for the numerical treatment of differential equations of large size, especially in the presence of stiffness.

In particular, exponential integrators turn out to be efficient when applied to partial differential equations; a common strategy is, indeed, to first discretize the problem along the spatial variables and hence solve the resulting large–size system of differential equations with respect to the time variable.

By means of exponential integrators the linear, and usually stiff, part of the system is solved exactly by a matrix exponential and explicit time–step methods are applied just to the remaining non stiff terms, thus to overcome stability issues with a reasonable computational effort.

The main topic of this talk is to discuss the generalization of exponential integrators to fractional–order problems and investigate their effectiveness when employed for the numerical treatment of partial differential equations with fractional time–derivatives.

The main theoretical aspects are investigated; we also discuss in detail some computational issues such as the evaluation of the action on some vectors of Mittag–Leffler functions with matrix arguments: this challenging task plays a fundamental role in the derivation of exponential integrators for fractional–order problems and requires the development of well–suited techniques.

Some numerical experiments illustrating the application of exponential integrators to some fractional PDEs are also presented.