Adaptive Finite Difference Methods with Variable Timesteps for Fractional Diffusion and Diffusion-Wave Problems

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One of the main drawback of finite difference methods when applied to fractional differential equations is that, due to the non-local nature of the fractional operators, they are slow and hugely memory demanding. Typically, the number of operations required to calculate the numerical solution at the n-th timestep scales as n^2, in sharp contrast with the linear growth for normal (non-fractional) equations. One way to alleviate this problem is by resorting to algorithms with variable timesteps, which allows one to use adaptive methods in which the size of the timesteps is chosen according to the behavior of the solution: large timesteps are used whenever the solution changes smoothly, whereas small ones are employed when the solution changes quickly. A minimum requirement of the numerical algorithm is that must be stable for the timesteps employed (ideally, it should be stable for any size of the timesteps). The criterium for choosing the timesteps depends on the problem one wants to solve, but a relatively general and robust procedure consists in to dynamically adjust their size so that the numerical error is smaller than a prefixed value. Two adaptive methods are considered and applied to some fractional diffusion and diffusion-wave problems. Their accuracy and speed are discussed.