Initial data transients in binary black hole evolutions

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Introduction

It is well known in numerical relativity that current practice for setting initial data introduces spurious radiation into the system, in both the 3+1 and the characteristic approaches. This leads to an initial burst of “junk” radiation. Common practice regards the signal as physical only after it has settled down following this burst. While it is straightforward to handle the junk burst in this way, a more serious issue is whether initial data errors lead to longer-term transients in the wave signal. **In this work we show that the initial data commonly used for both 3+1 and characteristic evolutions, leads to transients in the wave signal that last for several hundred M after the burst of junk radiation has passed.**

Characteristic extraction is a method of invariantly measuring gravitational wave emission by transporting the data to null infinity ($\mathcal{J}^+$). We first perform a 3+1 evolution, saving data on a worldtube at, say, $100M$. Subsequently, this data is used as innner boundary data for a characteristic evolution. Initial data is needed on a null cone in the far field region. Previous work has mainly taken the simplistic and unphysical approach of setting the null shear $J = 0$ everywhere.

Since characteristic initial data is needed only in the far field region, linearized theory provides a suitable approximation. Thus the key idea is to use the worldtube boundary data to construct initial data that, at the linearized approximation, represents the physical situation of purely outgoing radiation. We are then able to compare the waveforms computed by characteristic extraction using as initial data (a) $J = 0$, and (b) the linearized solution. We find that while the choice of initial J has only a small effect, a residual difference is visible, and takes several hundred M to be damped below other effects.

Any mis-match between the linearized solution and the actual data is an indication of an ingoing radiation content. Now, *on the worldtube* Γ, the characteristic metric data is determined entirely by the 3+1 data so that any mis-match can be traced back to the 3+1 initial data. In this way we show that the 3+1 initial data also contains a long-lasting transient.

The Bondi-Sachs metric

We start with coordinates based upon a family of outgoing null hypersurfaces. Let u label these hypersurfaces, x^A $(A = 2, 3)$ label the null rays, and r be a surface area coordinate. In the resulting $x^\alpha = (u, r, x^A)$ coordinates, the metric takes the Bondi-Sachs form

$$\begin{aligned} ds^2 &= -\left(e^{2\beta}(1 + W_c r) - r^2 h_{AB} U^A U^B\right) du^2 \\ &\quad -2e^{2\beta} du dr - 2r^2 h_{AB} U^B du dx^A + r^2 h_{AB} dx^A dx^B, \end{aligned} \quad (1)$$

where $h^{AB}h_{BC} = \delta^A_C$ and $det(h_{AB}) = det(q_{AB})$, with q_{AB} a unit 2-sphere metric. We represent q_{AB} by means of a complex dyad, and then, h_{AB} and U^A can be represented by complex numbers J and U respectively, with the spherically symmetric case characterized by $J = U = 0$.

Solutions to the linearized Einstein equations

It will be necessary to decompose the angular part of the metric quantities into basis functions, and for this purpose we use spin-weighted spherical harmonics ${}_sY_{\ell m}$. The standard spherical harmonics correspond to the case $s = 0$, and in this case the s will be omitted i.e. $Y_{\ell m} = {}_0Y_{\ell m}$. Quantities with $s = 0, 1$, and 2 are scalars, vectors, and 2-tensors on the 2-sphere.

Using the ansatz

$$F(u, r, x^A) = \Re(f_{\ell,m}(r) \exp(i\nu u))\, {}_sY_{\ell,m} \tag{2}$$

for a metric coefficient F with spin-weight s, we consider the lowest order case $\ell = 2$, and describe that part of the solution that represents purely outgoing gavitational radiation.

$$\beta_{2,\nu}(r) = b_1 \text{ (constant)} \tag{3}$$

$$j_{2,\nu}(r) = (12b_1 + 6i\nu c_1 + i\nu^3 c_2)\frac{\sqrt{6}}{9} + \frac{2\sqrt{6}c_1}{r} + \frac{\sqrt{6}c_2}{3r^3} \tag{4}$$

$$\begin{aligned} u_{2,\nu}(r) &= \sqrt{6}(\frac{\nu^4 c_2 + 6\nu^2 c_1 - 12i\nu b_1}{18} + \frac{2b_1}{r} + \frac{2c_1}{r^2} \\ &\quad -\frac{2i\nu c_2}{3r^3} - \frac{c_2}{2r^4}) \end{aligned} \tag{5}$$

$$\begin{aligned} w_{2,\nu}(r) &= r^2\frac{12i\nu b_1 - 6\nu^2 c_1 - \nu^4 c_2}{3} \\ &\quad + r\frac{-6b_1 + 12i\nu c_1 + 2i\nu^3 c_2}{3} + 2\nu^2 c_2 \end{aligned}$$

$$-\frac{2i\nu c_2}{r} - \frac{c_2}{r^2} \tag{6}$$

The solution is determined by setting the constant (real valued) parameters b_1, c_1 and c_2. The gravitational news corresponding to this solution is given by

$$\mathcal{N} = \Re(n_{2,\nu}\,\text{exp}(i\nu u))\,{}_2Y_{2,m} \;\text{ with } n_{2,\nu} = -i\nu^3 c_2 \frac{\sqrt{6}}{6} \tag{7}$$

Constructing the metric from data on a worldtube

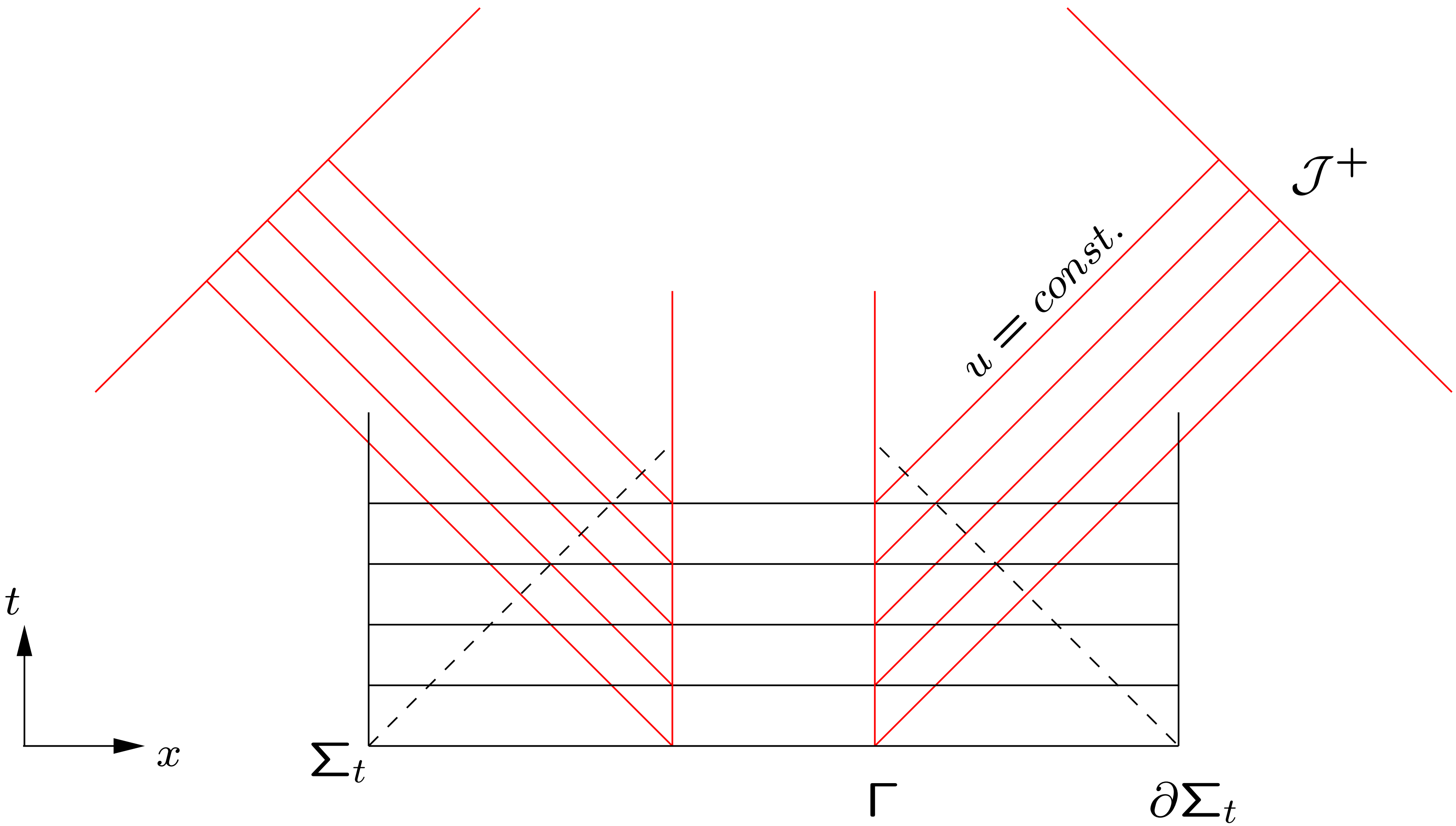
The Cauchy evolution provides the characteristic metric variables on the worldtube, decomposed into spherical harmonics ${}_sY_{\ell,m}$. We find coefficients of the linearized solutions that provide a fit to the numerical data at the worldtube. Then we use the linearized solutions with these coefficients to predict J everywhere at some chosen time u, and in this way provide initial data for a numerical characteristic evolution. We restrict attention to the dominant $2,2$ mode. The method uses a Fourier decomposition in the time domain, but since the details are technically complicated, they are not given here but can be found in the paper.

For each value of ν the linearized equations evaluated at the worldtube are four equations for the three unknowns $b_{1,\nu}, c_{1,\nu}, c_{2,\nu}$. Such an over-determined system can be tackled by ignoring one of the equations. We found a better fit to the actual data at $\mathcal{J}^+$ when the equation for W_c was ignored, so that a comparison between the actual and reconstructed data for W_c at the worldtube provides an indication of the error.

Numerical results

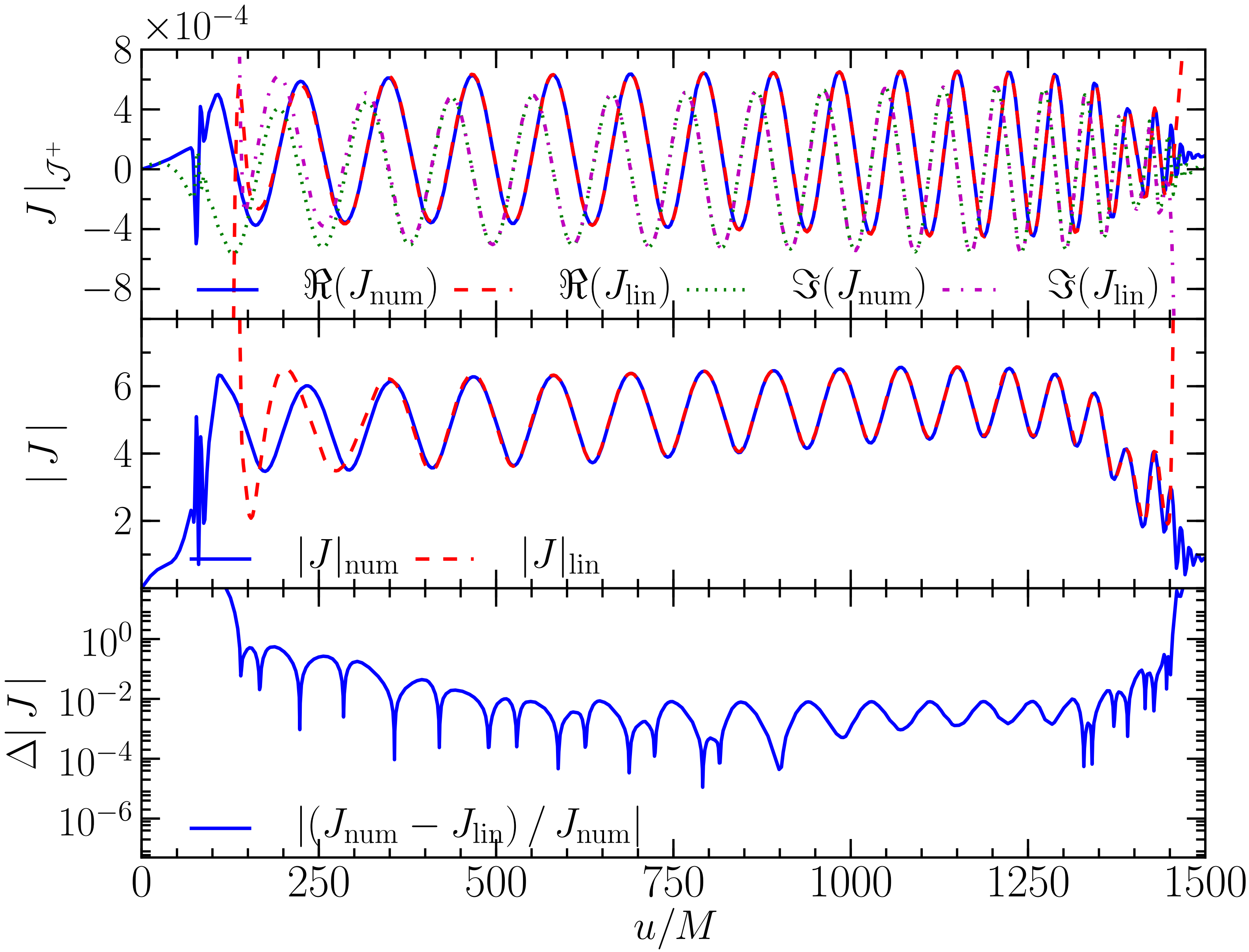
We use the well-studied model of an 8-orbit binary system with equal mass non-spinning black holes. For the Cauchy evolution, we use the `Llama` multipatch code. We output metric data on a worldtube located at $R_\Gamma = 100M$. The Table below summarizes the various characteristic evolutions that we have performed, all of which are based on the same Cauchy data, but with different characteristic initial data, J, and starting points in Bondi time, u_0.

Data set	Worldtube location $R_\Gamma[M]$	Initial time $u_0[M]$	Initial data J
J0-R100-u0	100	0	$J = 0$
J0-R100-u450	100	450	$J = 0$
Jlin-R100-u450	100	450	$J = J_{\rm lin}$



$\mathcal{J}^+$
$u = const.$
t
x
Σ_t
Γ
$\partial\Sigma_t$

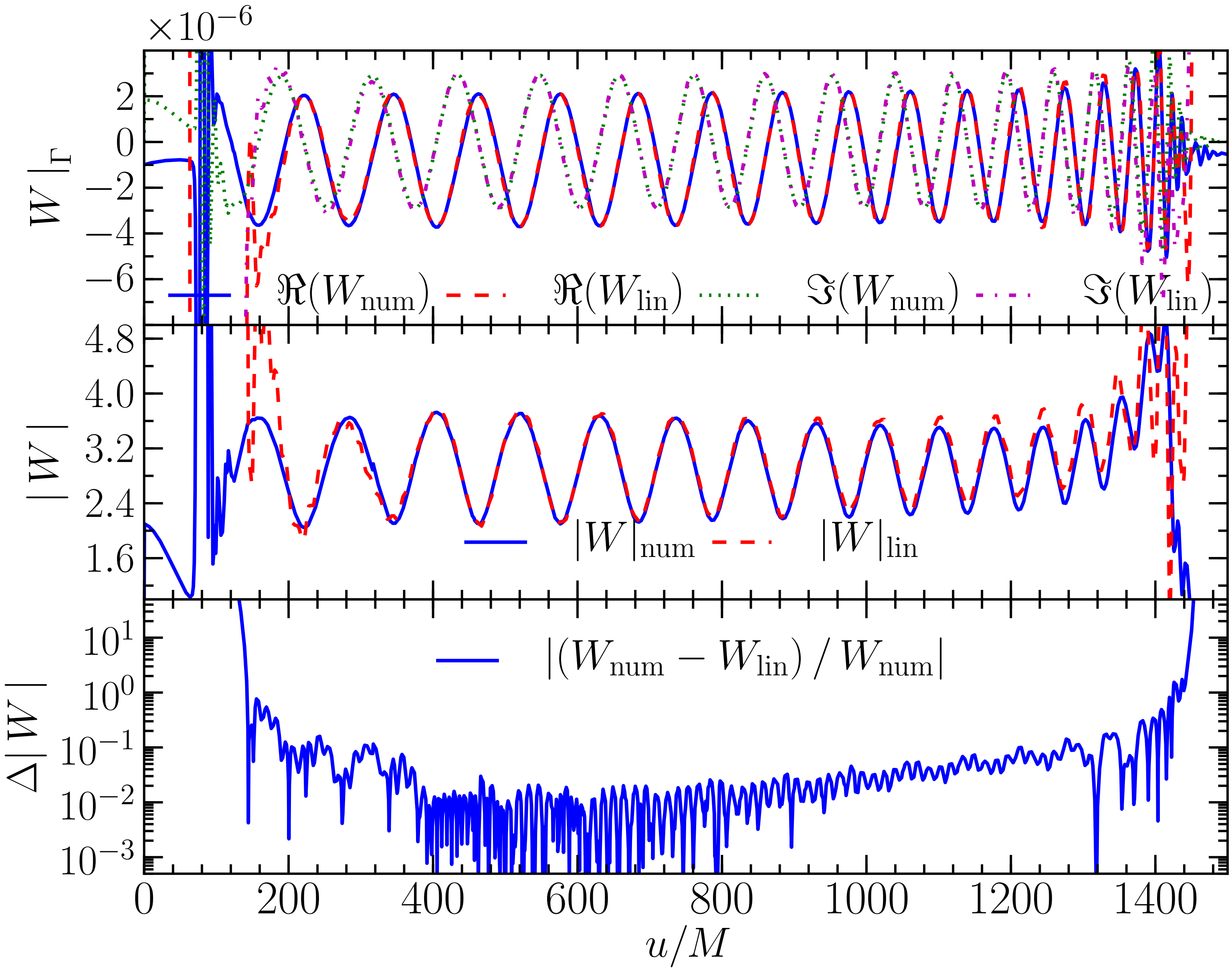
In Model J0-R100-u0, the characteristic evolution is started using the first available Cauchy data, at coordinate time $t_0 = u_0 = 0$, and initialized by the shear-free solution $J = 0$. The characteristic evolution includes the spurious junk radiation. We plot the $(\ell, m) = (2, 2)$ modes of J_{num} computed by J0-R100-u0, and compare them to those of the linearized solution J_{lin} computed using linearly reconstructed worldtube data. The upper panel plots the real and imaginary parts of J, evaluated at $\mathcal{J}^+$. The center panel plots the amplitudes of J, while the bottom panel shows the relative difference between the linearly estimated J_{lin} and J_{num}. The linearized J_{lin} and numerically evolved J_{num} differ initially, but from about $u = 450M$ differ by less that 1%. The difference between J_{lin} and J_{num} is caused by spurious radiation in either the characteristic, or the 3+1, initial data, or both.



$\times 10^{-4}$
$J|_{\mathcal{J}^+}$
$\Re(J_{\rm num})$
$\Re(J_{\rm lin})$
$\Im(J_{\rm num})$
$\Im(J_{\rm lin})$
$|J|$
$|J|_{\rm num}$
$|J|_{\rm lin}$
$\Delta|J|$
$|(J_{\rm num} - J_{\rm lin}) / J_{\rm num}|$
u/M

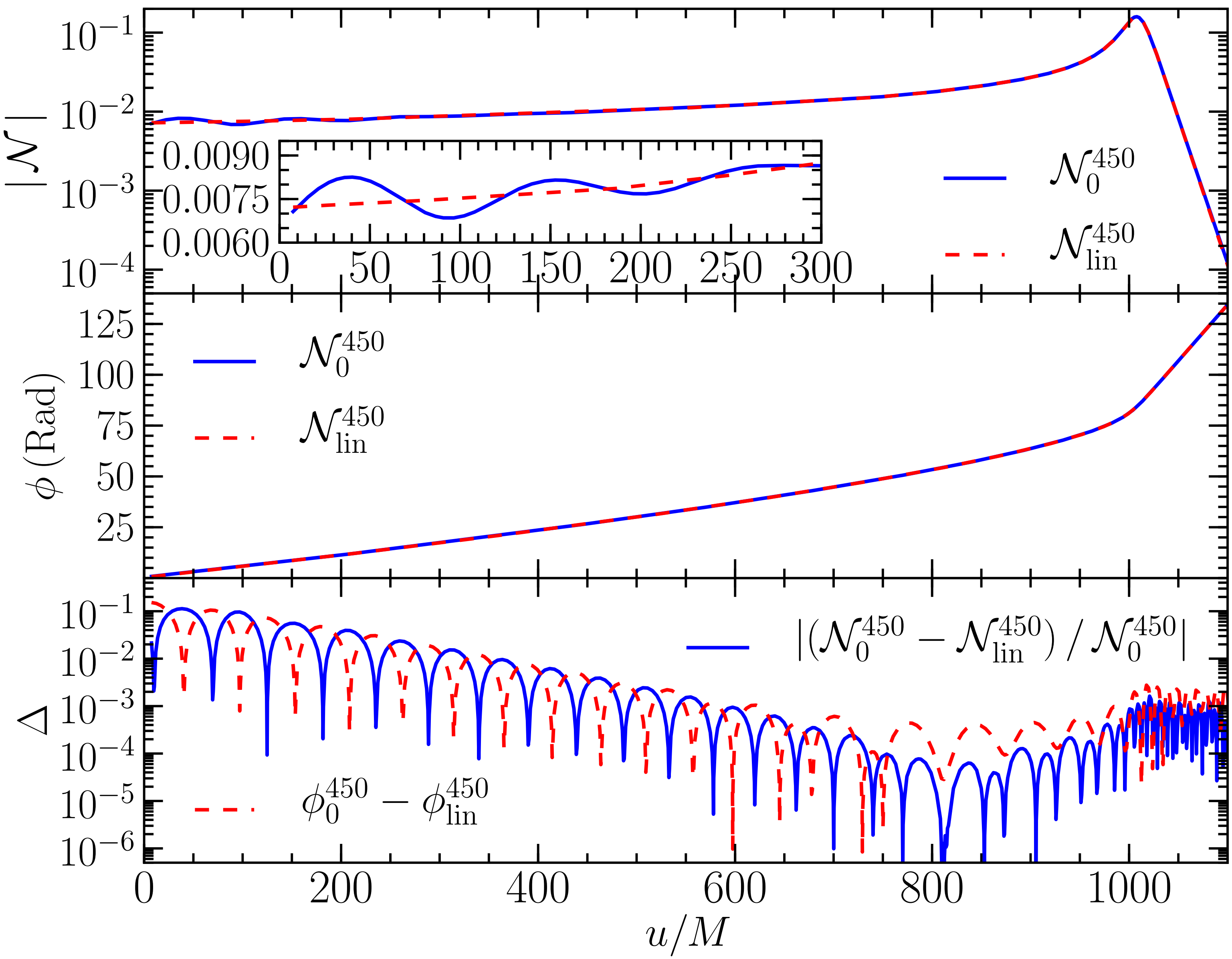
A similar effect is seen in the characteristic variable W_c, plotted overleaf. In this case there is clarity about the source of the incoming radiation: it must be in the Cauchy data. This is because in characteristic extraction, the characteristic metric at the worldtube is determined entirely by the Cauchy data. Again, the lower panel shows an approximately exponential decay in the differences, until around $u = 400M$.

The findings above indicate that a physically expected purely outgoing inspiral radiation pattern is present after some time $u_0 > u_{\text{incoming}}$, and it is only after this time that it is possible to use the linearized solutions to construct physically consistent initial data. The above results suggest $u_{\text{incoming}} \approx 450M$.



$\times 10^{-6}$
$W|_{\Gamma}$
$\Re(W_{\rm num})$
$\Re(W_{\rm lin})$
$\Im(W_{\rm num})$
$\Im(W_{\rm lin})$
$|W|$
$|W|_{\rm num}$
$|W|_{\rm lin}$
$\Delta|W|$
$|(W_{\rm num} - W_{\rm lin})\,/\,W_{\rm num}|$
u/M

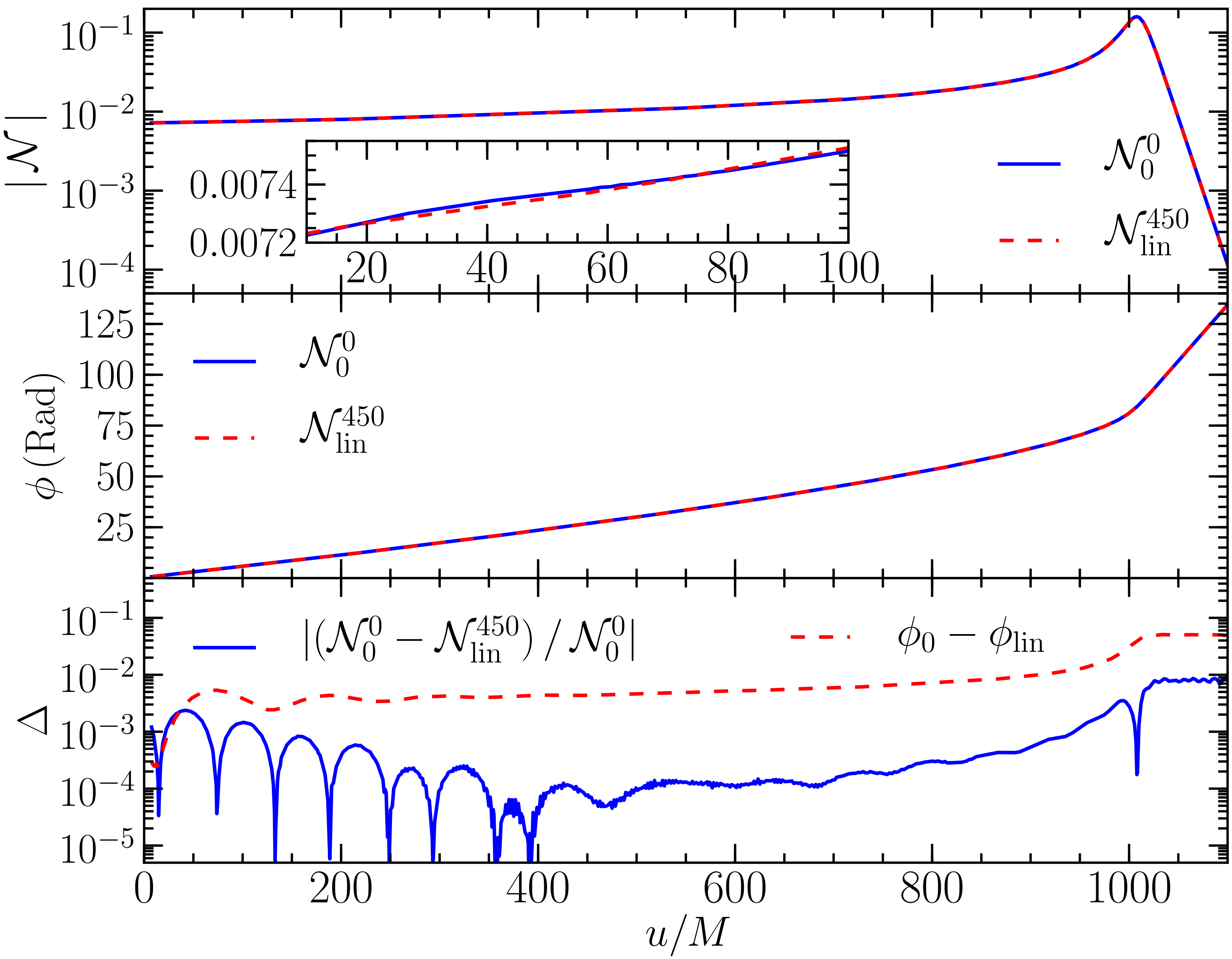
We compare waveforms at $\mathscr{I}^+$ from two characteristic evolutions based on the same Cauchy boundary data, but different initial data: Jlin-R100-u450 and J0-R100-u450. Both evolutions use boundary data from $R_\Gamma = 100M$ and begin at $u_0 = t_2 = 450M$. The model Jlin-R100-u450 uses initial data determined by the linearized solution, whereas J0-R100-u450 sets $J = 0$. The plots are overleaf, with the evolutions denoted by $\mathcal{N}^{450}_{\rm lin}$ and $\mathcal{N}^{450}_0$. Whereas the phase shows very little difference between the runs (middle panel), the amplitude shows visible oscillations for the $\mathcal{N}^{450}_0$ evolution (upper panel and inset). The waveforms agree to within 1% only after a time $u = 400M$ (which must be added to the $u_0 = 450M$ starting point of the simulation).



$|\mathcal{N}|$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0.0090
0.0075
0.0060
0
50
100
150
200
250
300
$\mathcal{N}_0^{450}$
$\mathcal{N}_{\rm lin}^{450}$
$\phi\,(\mathrm{Rad})$
125
100
75
50
25
$\mathcal{N}_0^{450}$
$\mathcal{N}_{\rm lin}^{450}$
Δ
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$\left|\left(\mathcal{N}_0^{450} - \mathcal{N}_{\rm lin}^{450}\right)/\,\mathcal{N}_0^{450}\right|$
$\phi_0^{450} - \phi_{\rm lin}^{450}$
0
200
400
600
800
1000
u/M

We also compare the waveforms of model Jlin-R100-u450 against those of model J0-R100-u0 (which uses $J = 0$ at $u_0 = 0$) labeled by $\mathcal{N}_0^0$. We still observe an oscillation in the amplitude in $\mathcal{N}_0$ (but it is drastically reduced compared to the $\mathcal{N}_0^{450}$ of model J0-R100-u450 shown above). The relative errors in amplitude, are well below 1% over the entire evolution, and the total dephasing is smaller than $\Delta\phi = 0.04\mathrm{rad}$.

The numerical tests described so far were with the extraction radius $R_\Gamma = 100M$. All these runs were repeated with the extraction radius re-set to $R_\Gamma = 250M$. The results are qualitatively similar to the $R_\Gamma = 100M$ case.



$|\mathcal{N}|$
$\mathcal{N}_0^0$
$\mathcal{N}_{\rm lin}^{450}$
$\phi\,(\mathrm{Rad})$
$|(\mathcal{N}_0^0 - \mathcal{N}_{\rm lin}^{450})\,/\,\mathcal{N}_0^0|$
$\phi_0 - \phi_{\rm lin}$
Δ
u/M

Discussion and Conclusion

The linearized gravitational wave solution is compatible with outgoing radiation. As such, it provides a more physically motivated starting point than the shear-free, $J = 0$, alternative. Importantly, we find that the evolutions which take place from either $J = J_{\text{lin}}$ initialized at a time $u_0 = 450M$, and $J = 0$ at the initial time $u_0 = t_0 = 0$, are very similar. That is, for simple choices of initial J, the physical conclusions are not altered dramatically.

On the other hand, we have demonstrated that the choice of characteristic initial data does result in a small but measurable difference, which decays at a slow exponential rate over a time period of several hundred M. Since the linearized initial data contains only an outgoing mode, we conclude that the shear-free characteristic initial data contains *incoming* radiation. While this is expected, it is interesting that it takes so much time for the effect to decay away.

Since the characteristic data on the worldtube is determined entirely by the 3+1 data, the extent to which this data does not fit the linearized solution is a measure of its incoming radiation content. By construction, the quantities β, U and J in the linearized solution must fit the data, with the difference in W_c being an indication of incoming radiation in the 3+1 evolution. Since W_c is not a gauge invariant quantity, it is not possible to make a quantitative statement about the magnitude of the incoming radiation, but our work indicates that it takes until at least $\pm 400M$ until the effect of incoming radiation is saturated by other effects.