On the merger of small and large black holes

Philipp Mösta[1], Lars Anderson[1], Jan Metzger[2], Bela Szilágyi[3], Jeff Winicour[4]

[1] Max-Planck-Institut für Gravitationsphysik, Albert-Einstein-Institut, 14476 Golm, Germany
[2] Mathematisch-Naturwissenschaftliche Fakultät, Universität Potsdam, Potsdam, Germany
[3] Theoretical Astrophysics, California Institute of Technology Pasadena, CA 91125, USA
[4] Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA 15260, USA

Motivation

An unexpected feature has been observed in the simulation of binary black holes following their inspiral and merger. [1] After formation of a common apparent horizon, as the two marginally outer trapped surfaces (MOTS) corresponding to the individual black holes continue to approach, they eventually touch and then penetrated each other. This penetration was surprising because it had not been considered in theoretical discussions [3, 4, 5] and had not been observed in prior simulations of binary black hole mergers. In retrospect, it is tempting to speculate in some heuristic sense that a small black hole should enter a very large black hole with hardly any notice of its presence. In fact, it has been been conjectured on the basis of the equivalence principle that a very small black hole should, in some appropriate sense, fall into the large one along a geodesic. However, such a perturbative picture is unreliable in the interior of the event horizon surrounding the two black holes, where the MOTS exist. In this study we use an harmonic evolution code to first confirm that the individual MOTS do penetrate following a binary black hole merger and then analyze some of the highly interesting features revealed by the simulations.

Setup and stages of evolution

We study a head-on unequal-mass black hole merger with a 1/4 mass ratio. The initial separation of the two individual black holes is 1M and we use time-symmetric puncture initial data. The black-holes have no angular momentum and we use a harmonic formulation of the Einstein equations presented in detail in [2]. There are 5 distinctive stages of interest in the simulations:

- Large separation - individual MOTS
- Formation of a common horizon as the MOTS approach
- Initial osculation of the two MOTS
- Penetration of the two MOTS
- The ultimate fate of the individual MOTS

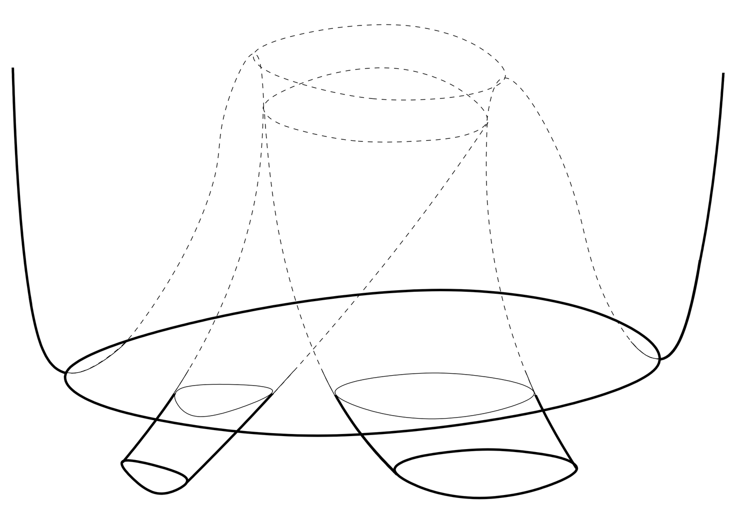

Figure 4: A possible scenario for the evolution of the trapped tubes. The outermost MOTS are drawn in bold.

Fig. 4 suggests a possible scenario for the evolution of trapped tubes in a binary black-hole merger. Initially both MOTS are well separated and their geometrical properties are purely determined by the choice of initial data. As the MOTS approach eventually a common apparent horizon will be formed. To be more precise a pair of two MOTS, a stable and an unstable branch is formed. The two surfaces evolve differently in time. The stable branch forms the outer common horizon and stays regular while the unstable branch shrinks and eventually seizes to exist. At the time the MOTS touch for the first time, their mean curvatures at these points have to agree. Then as they continue to overlap their geometrical properties are mostly determined by their interaction.

A MOTS $\mathcal{S}$ is found by searching for solutions to

$$\theta_\pm = P \pm H = 0,$$

where $\theta_\pm$ denotes the expansion,

$$H = s^{\mu\nu}\nabla_\mu N_\nu$$

is the mean curvature of $\mathcal{S}$ in the Cauchy slicing, and

$$P = s^{\mu\nu}\nabla_\mu n_\nu$$

is the 2-trace of the extrinsic curvature of the Cauchy slicing and ∇ is the covariant derivative with respect to the space-time metric g.

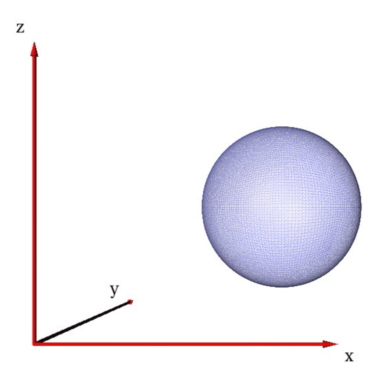

Figure 5: Individual apparent horizons after $t = 0.384M$ of evolution time.

Simulation results

Fig. 1 shows the coordinate representations of both individual apparent horizons and their mean curvature as a function of the horizon finders six-patch angular coordinate system at time $t = 1.452M$ (already after a common horizon has formed) and at $t = 1.974M$ (both MOTS touch for the first time). One can clearly see the tidal effect of the small MOTS on the mean curvature of the larger MOTS, which develops a local peak in its mean curvature located towards the small MOTS. This peak is smoothly growing in time until finally both mean curvatures agree at the point of osculation.

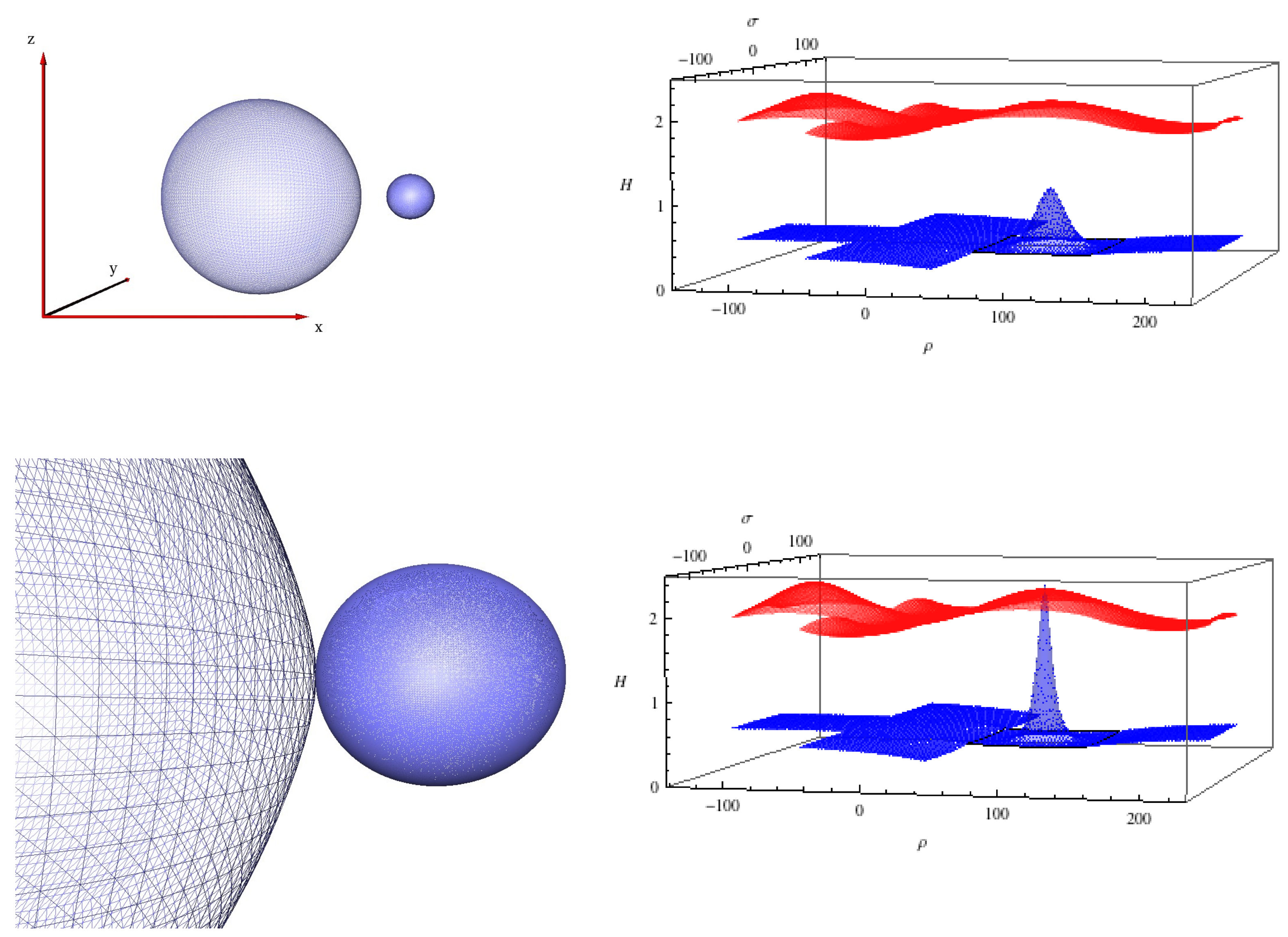

Figure 1: Coordinate representation of the apparent horizons (left panels) and mean curvature as a function of the coordinates of the horizon finders six-patch coordinate system (right) at times $t = 1.452M$ (top panel) and $t = 1.97M$ (bottom panel).

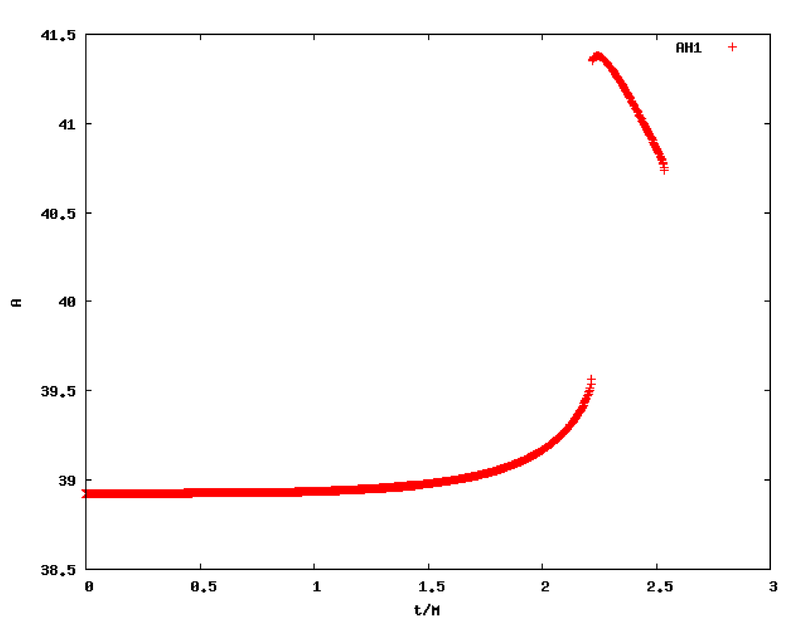

Figure 2: Time evolution of the area of the larger apparent horizon.

As the individual MOTS continue to overlap we observe another interesting feature in our simulations. As the puncture of the smaller MOTS approaches the surface of the larger MOTS we see a jump in the area evolution of the larger MOTS. This is not unexpected from mathematical theory, as the world tube of a MOTS can intersect a given slice more than once and it is possible to jump from one solution to the other. This jump can also be seen in the coordinate representations of the apparent horizons. The smaller MOTS does not show a jump in its area evolution but continues to stay fairly regular in shape and geometry.

Fig. 3 shows the coordinate shapes of both apparent horizons at a time $t = 2.538M$. After that we cannot find the larger MOTS anymore while the smaller one continues to exist. We can only speculate why we lose the ability to track the evolution of the larger MOTS. Either the MOTS becomes to irregular in its shape and the apparent horizon finder fails for numerical reasons or the MOTS becomes unstable and stops to exist. Both scenarios are possible but at the moment we can only speculate to which one is the case.

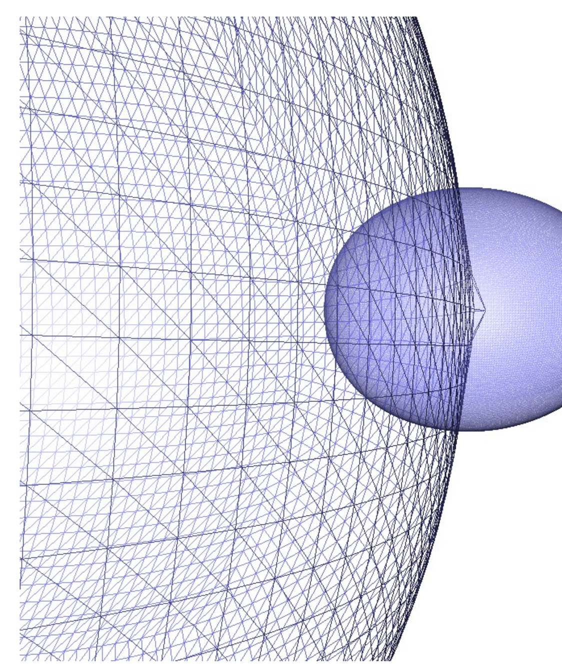

Figure 3: Intersecting individual apparent horizons at time $t = 2.538M$.

Discussion

- We have analyzed the geometrical properties of MOTS in a head-on unequal-mass binary black-hole merger with a mass ratio of $q = 1/4$.
- During the course of our simulations we have investigated the evolution of the geometry of both individual and common apparent horizons.
- While the two individual MOTS approach, there is a strong tidal effect of the small MOTS on the larger one, which is visible both in the coordinate representation and the mean curvature. Finally when both surfaces touch their mean curvatures agree as predicted by mathematical theory.
- We track the evolution of both individual MOTS until the smaller MOTS is more than half-way into the larger MOTS. At this point the puncture of the small MOTS might interfere with the geometry of the larger MOTS and how the detailed merger of the two surfaces goes about in the final stages of the evolution has yet to be determined.

References

[1] B. Szilágyi, D. Pollney, L. Rezzolla, J. Thornburg and J. Winicour, "An explicit harmonic code for black-hole evolution using excision", *Class. Quantum Grav.*, **24**, S275 2007.

[2] P. Mösta, "Puncture Evolutions within the Harmonic Framework", Diploma thesis, Universität Kassel (2008).

[3] S. A. Hayward, "General laws of black hole dynamics", *Phys.Rev. D* **49**, 6467 (1994).

[4] A. Ashtekar and B. Krishnan, "Dynamical horizons and their properties", *Phys.Rev. D* **68**, 104030 (2003).

[5] A. Ashtekar and B. Krishnan, "Isolated and dynamical horizons and their applications", *Living Rev. Rel.* **7**, 10 (2004).

Acknowledgements

We would like to thank Michael Koppitz for assisting on the visualization and Jose-Luis Jaramillo for insightful discussions.

Contact

Don't hesitate to contact

philipp.moesta@aei.mpg.de