High-accuracy simulations of extreme mass-ratio black hole binaries in the time-domain

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Collaborators & Support

- **Lior Burko, U Alabama Huntsville**
- **Scott Hughes' Group, MIT**
- **Students: Justin McKennon, Gary Forrester, Mike DeSousa (UMass Dartmouth)**

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Kerr (rotating) black hole perturbation theory

- **Write Einstein's GR field equations to linear order expanding about a BH solution**
- ***Teukolsky equation*** **-- a wave-equation like PDE that describes how generic fields (scalar, vector, tensor) in the space-time of a Kerr BH behave/evolve**
- **In the gravitational field context -- describes the behavior of GWs in Kerr space-time geometry**
- **Relatively simple:** ***linear, hyperbolic, (3+1)D PDE ..***

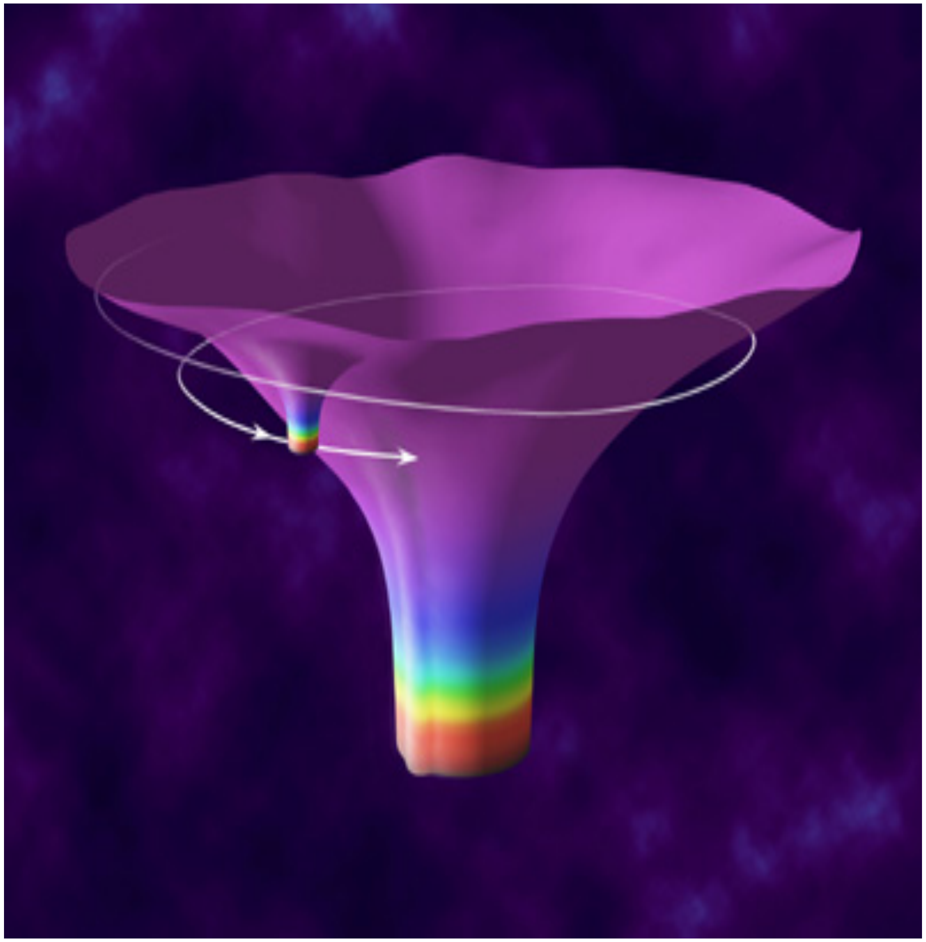
$$\left\{ \left[a^2 \sin^2\theta - \frac{(r^2+a^2)^2}{\Delta}\right]\partial_{tt} - \frac{4Mar}{\Delta}\partial_{t\varphi} - 2s\left[(r+ia\cos\theta) - \frac{M(r^2-a^2)}{\Delta}\right]\partial_t \right.$$

$$+\Delta^{-s}\partial_r\left(\Delta^{s+1}\partial_r\right) + \frac{1}{\sin\theta}\partial_\theta\left(\sin\theta\partial_\theta\right) + \left[\frac{1}{\sin^2\theta} - \frac{a^2}{\Delta}\right]\partial_{\varphi\varphi}$$

$$\left. +2s\left[\frac{a(r-M)}{\Delta} + \frac{i\cos\theta}{\sin^2\theta}\right]\partial_\varphi - s\left(s\cot^2\theta - 1\right)\right\}\Psi = 4\pi\Sigma T,$$

EMRIs : extreme-mass-ratio black hole binary inspirals

- **What happens when a small compact object is captured by such a *supermassive* black hole?**
- **The *ESA/NASA LISA Mission* will detect GWs from such a process**
- **EMRIs also "map" the spacetime of the central hole and thus may provide key theoretical insights ..**
- **LISA data analysis requires theory-based waveforms with *relative errors less than 0.01% (!)***



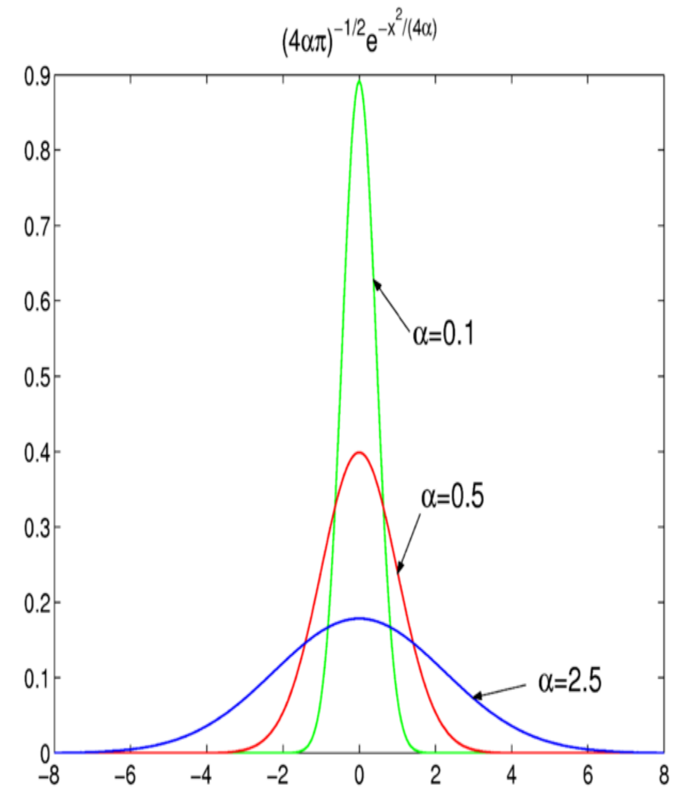
High-accuracy approach

- ***Mathematical advancements*****: improvements to representation of point-particle on numerical grid; exploration of pseudo-spectral method, higher-order finite-difference methods and hybrid-methods**
- ***Standard extrapolation methods*****: extrapolation to infinite extraction radius and (Richardson) extrapolation infinite grid density**
- ***Novel & advanced HPC hardware*****: code performance enhancements using GPUs for "brute-force" improvements in the results**

Resulting outcome: Can now generate long waveforms (several 100,000M) with relative errors less than 0.01% in just a few hours!

Math. Adv. : Point-like object

- **Source of the GWs (perturbation) in EMRI is the inspiraling object**
- **How to model a point-like compact object (technically a *Dirac-delta* function) on a numerical grid?**
- **Obvious approach would be use a narrow *Gaussian* distribution; do several runs with successively narrower profiles & take a limit ..**
- **Decent results, but very expensive**
- **Alternative approach is to develop a *discrete-delta function on numerical finite-difference grid***
- **Extremely efficient (by over an *order-of-magnitude*!)**

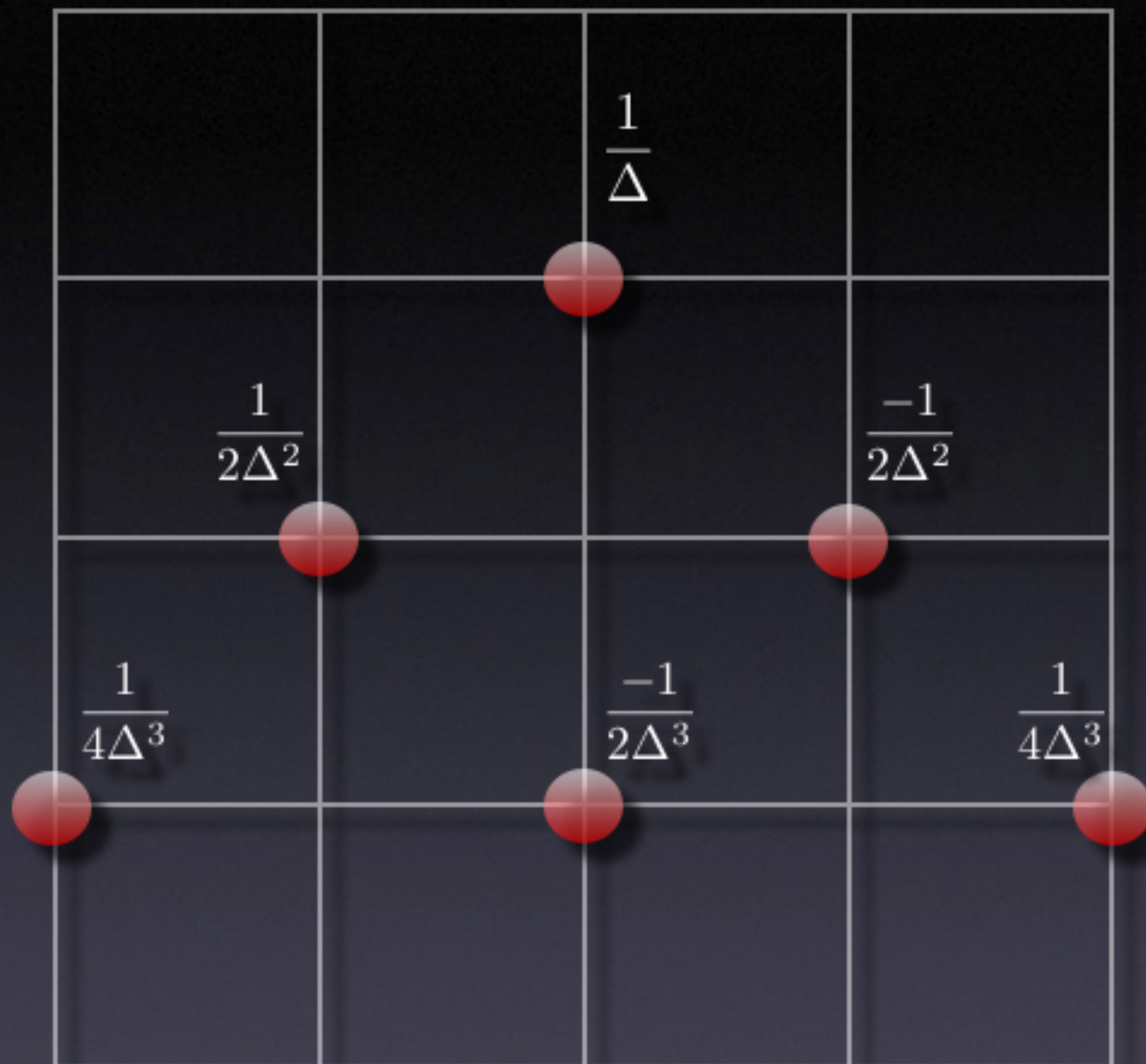


[*Sundarajan, GK, Hughes: Phys. Rev. D76, 104005 (2007)*]

$\delta(x)$

$\delta'(x)$

$\delta''(x)$



Δ : grid spacing

Math. Adv. : Hyperbolic PDE solve

- **Can decompose Teukolsky equation in azimuthal m-modes in the time-domain**
- **(2+1)D system of linear, hyperbolic PDEs**
- **Rewrite this system in 1^{st} order PDE form**
- **Discretize on a 2D spatial r* & theta grid**
- **Current Teukolsky code is a *time-explicit, 2-step Lax-Wendroff finite-difference* solver**
- **2^{nd}-order in time & radial r* grid operations**
- **4^{th}-order in theta (polar angle) grid operations (generate a much higher truncation error!)**
- ***Alternatively multi-domain pseudo-spectral* method works extremely well for the homogeneous PDE solver (Chebyshev collocation grid in r*; Legendre in theta)**
- **Challenging to use with the singular sources!**

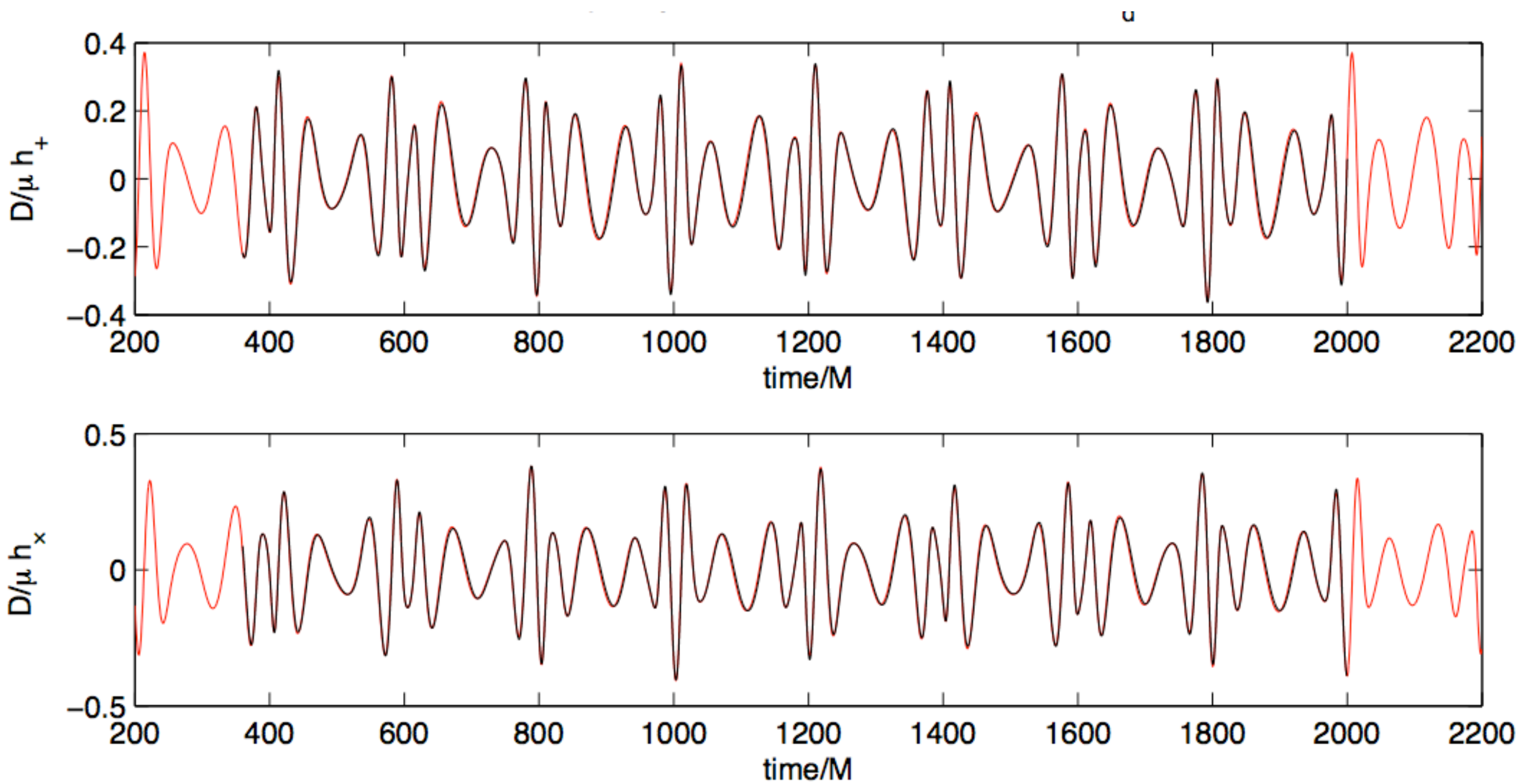
HPC Adv. **: GPUs & Cell BE**

- **Exploit parallelism of *many-core* processor architectures for brute-force gains in performance and accuracy**
- **Nvidia *CUDA "Fermi" GPUs* and PS3's *Cell Broadband Engine***
- **Easily obtain *10x overall performance gain* in double-precision over 8-core x86 CPUs**
- **Excellent scaling on such clusters observed as well**
- **See Justin McKennon's poster for more details …**

Results: Fluxes (time-domain)

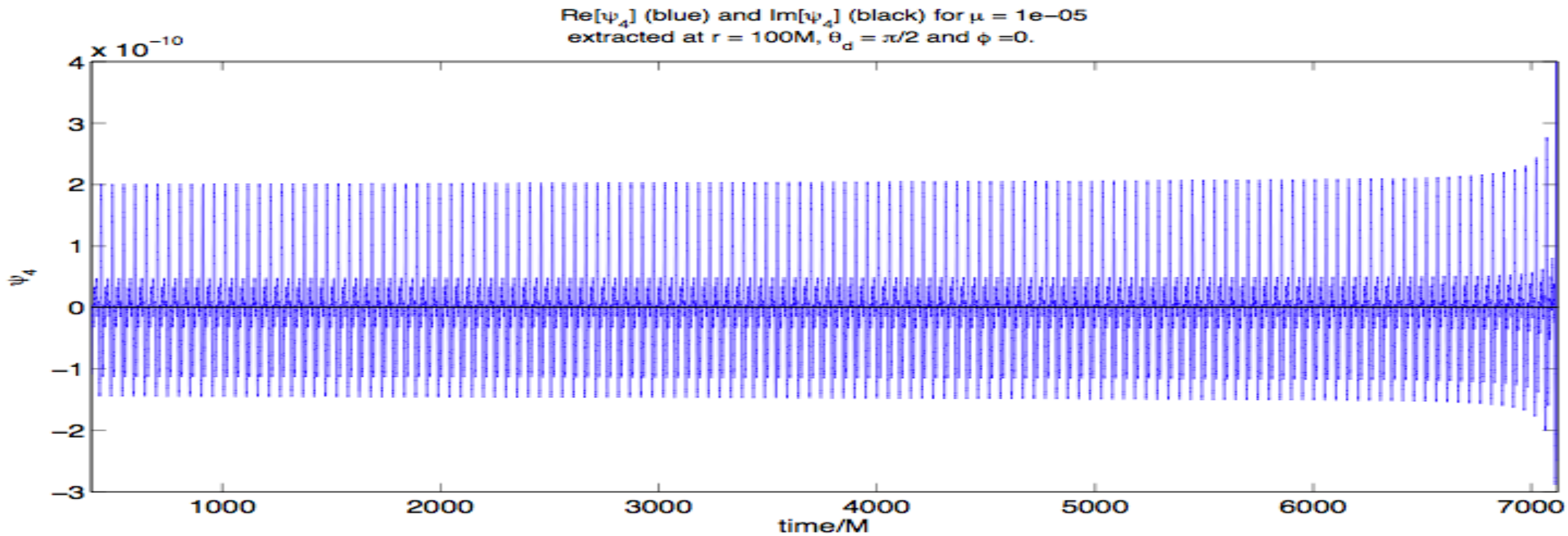
- **Compute *energy, angular momentum and linear momentum fluxes* directly from Psi_4**
- **Compare energy fluxes from well-established values obtained from the frequency-domain for a variety of *perfectly circular orbits* for a number of black hole spins**
- **Agreement *better than 1 part in 10,000!***
- **Resolutions used: dr*/M = dtheta/rad = 0.01**
- **Evolution is a few thousand M long**
- **Extraction radii r* = 100M, 200M, 300M, 400M, 500M, 600M, 700M and 800M**
- **CUDA GPU code takes only a few hours to run!**

Results: Waveforms (frequency-domain comparison)

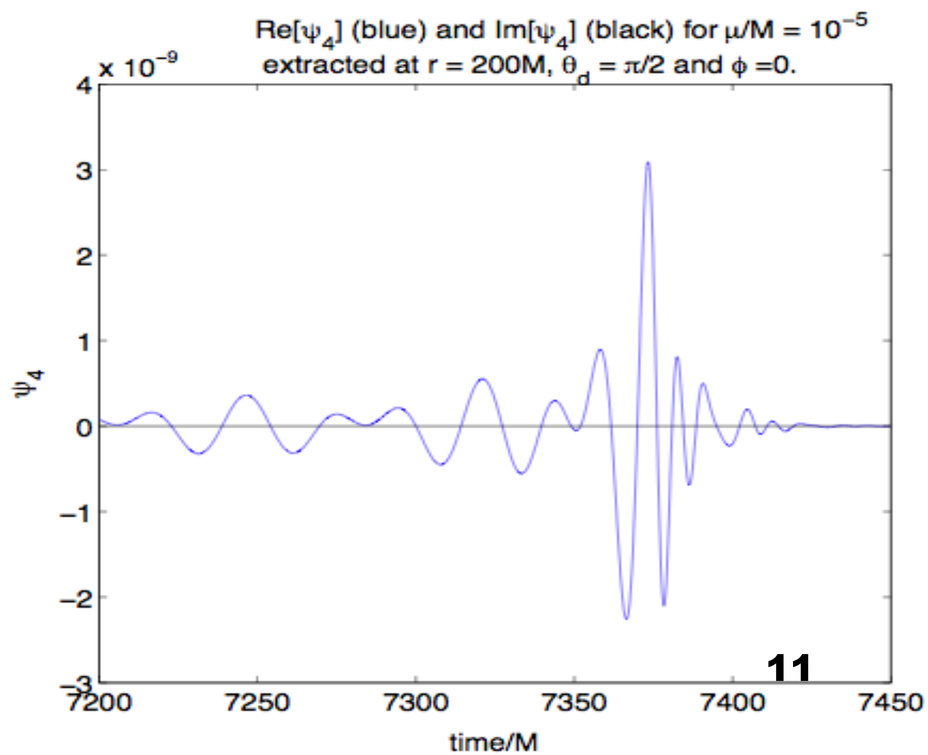


$p = 6M$, $e = 0.3$ and $\theta_{\rm inc} = \pi/3$ about a black hole with spin $a/M = 0.9$

[***Sundarajan, GK, Hughes, Drasco: Phys.Rev. D78, 024022, 2008***]

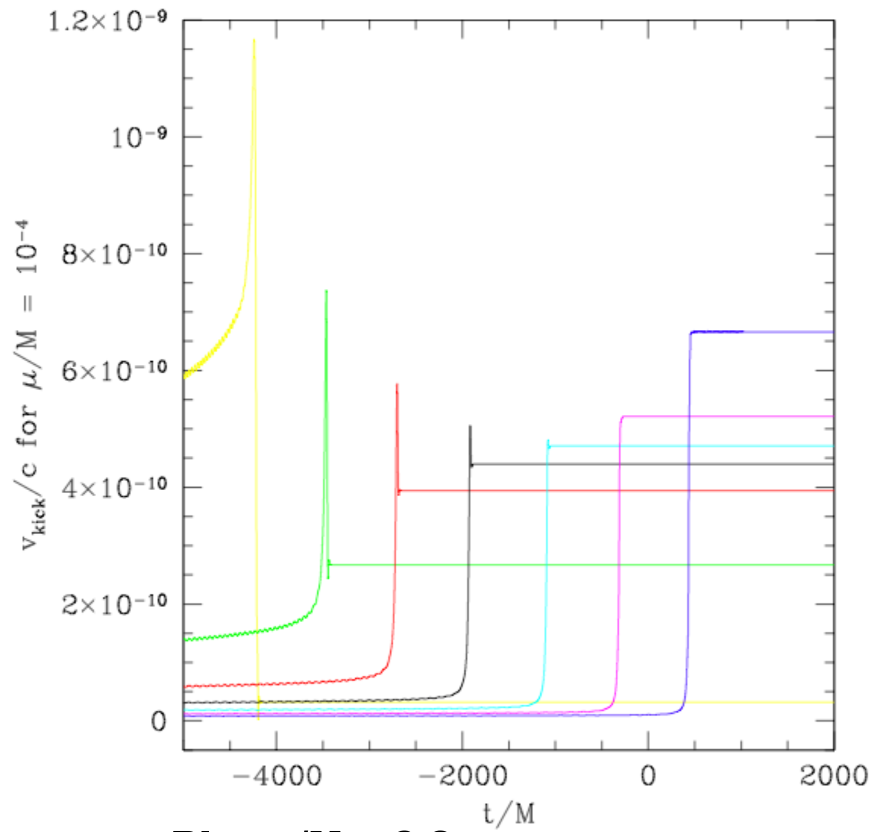


Complete inspiral using a *hybrid* approach: time-domain for waveforms & frequency-domain for orbital decay details



Results: Recoil velocities ("kick")

- **Compute *linear momentum fluxes* directly from Psi_4**
- **Balance will yield an estimate of the GW "kick"**
- **Study "kick" as a function of spin for pro- and retro-grade inspiralling orbits**
- **Accurate estimates require long evolutions and multiple m-modes!**
- **Of particular interest is the nature of the "anti-kick" for the high-spin prograde case**



Blu a/M= -0.9
Mag a/M= -0.6
Cyn a/M= -0.3
Blk a/M= 0.0
Red a/M= 0.3
Grn a/M= 0.6
Ylw a/M= 0.9

[*Sundarajan, GK, Hughes: Phys. Rev. D81, 104009 (2010)*]