Constructing more astrophysically realistic binary black hole initial data

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Abstract

As numerical simulations of black hole binaries continue to mature, the construction of astrophysically realistic initial data for these simulations remains an important challenge. In particular, such initial data will likely be needed for the simulations that will supply advanced detectors with templates for parameter estimation, since current binary black hole initial data clearly do not correspond to a timeslice of an astrophysical black hole binary: All sets lead to an initial burst of spurious ("junk") radiation when evolved, and all but two of the sets lack the outgoing radiation from the binary's past inspiral. We discuss various methods of constructing more astrophysically realistic initial data, and, in particular, describe the data we have generated by matching the post-Newtonian (PN) metric to two perturbed Schwarzschild metrics. These data include the binary's outgoing radiation, in addition to the expected tidal deformations on the holes (whose absence in standard initial data is thought to be the cause of the high-frequency component of the junk radiation). Our matched data are currently being evolved in two different implementations, and we discuss ways of improving the construction. We also investigate how much the spurious radiation (and lack of initial outgoing radiation) in simulations affects the binary's subsequent evolution through post-Newtonian tail effects.

1. Motivation

What do we mean by astrophysically realistic binary black hole initial data?

We mean initial data that describe a timeslice of an astrophysical black hole binary with sufficient accuracy that the subsequent evolution is indistinguishable from the evolution of the true, astrophysical system to the accuracy required of the simulation. (This accuracy could be set, for instance, by the requirement that the resulting waveform be accurate enough for parameter estimation with a given gravitational wave detector at some large SNR—cf. [1, 2].)

Of course, there are idealizations: We consider an isolated binary, and take the common case of one that has shed (almost all of) its eccentricity during a long inspiral.

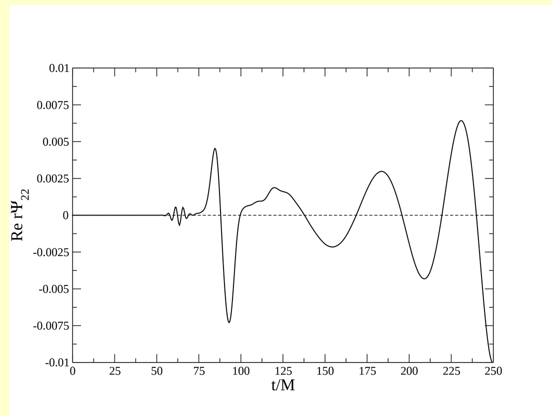

Fig. 1: Beginning of a waveform from [3] (puncture data, equal-mass nonspinning, with an initial PN separation of $\sim 6.79m$).

How well do current initial data describe such spacetimes?

The spurious radiation (see Fig. 1) is an indication of a lower limit to such data's accuracy (and causes some problems itself for SpEC, and in all codes from tail effects), as are recent reports of incoming radiation (see [4] and N. Bishop's poster), but we can also identify some specific desiderata.

(High) spin and (low) eccentricity: Many astrophysical black hole binaries are expected to have high spins and low eccentricities close to merger.

Current superposed data sets can give high spins [5] ($\lesssim 0.999$, with evolutions with spins up to 0.95 [6]), and iterative methods yield eccentricities around 10^{-5} [7] ($< 10^{-4}$ with spin [8]); the canonical PN prediction for the eccentricity is $\sim 10^{-6}$ [9].

(Appropriate) radiation content: The initial data should contain all the outgoing radiation from the binary's long inspiral, and no other radiation (in particular, no incoming radiation).

Only the Kelly *et al.* data [10] and our matched data include the binary's outgoing radiation. (Both use PN results to do so.) The initial data should also include the expected tidal deformations on the holes—the lack of these is thought to lead to the high-frequency component of the junk radiation. The tidal deformations are only explicitly included in the matched initial data. (And, indeed, while Kelly *et al.* see some reduction in the initial junk in their evolutions [11, 12], the high-frequency component remains unchanged.)

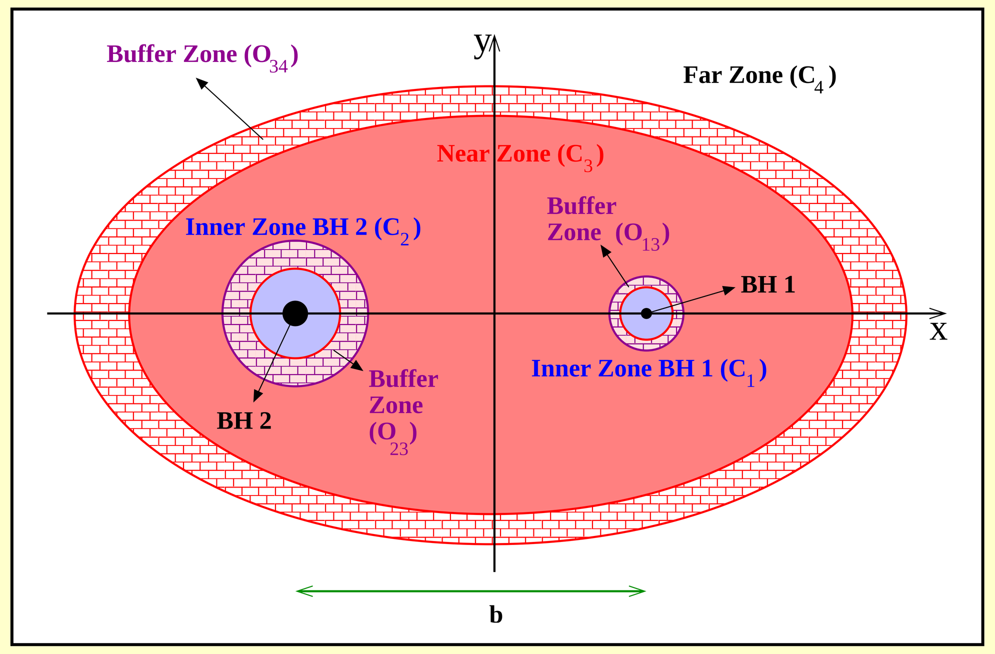

Fig. 2: Illustration of the zones

2. Our matched construction

We have carried out the matching construction first considered in [13, 14, 15] to the highest order possible using the results available at the time, so our data are *conformally curved* and include the binary's outgoing radiation, in addition to the tidal deformations on the holes.

Basic idea of our construction: In each of the zones shown in Fig. 2, there is a metric that is a good approximation to the binary's true metric in that zone. Moreover, the zones overlap, allowing us to fix any undetermined parameters (including coordinate transformations) using the method of **matched asymptotic expansions**.

We have considered the simplest possible case of a nonspinning binary in a quasicircular orbit, so we use the following metrics:

Inner zones: Perturbed Schwarzschild metrics from [16], converted to (horizon-penetrating) Cook-Scheel harmonic coordinates [17].

Near zone: Standard PN metric from [18] (with a perturbative treatment of retardation).

Far zone: Retarded PN metric from [19], with a multipole expansion in {size of the source}/{distance}, and the binary's past history computed using the highest-order PN results available (from [9]).

The zones overlap in so-called *buffer zones* where the approximations used to obtain two of the metrics are both valid. We use this formal agreement (of bivariate power series) to read off the tidal perturbations of the holes and put the inner zone metrics in the same coordinate system as the PN metric (to the order we have matched). We also stitch the metrics together numerically in these buffer zones using freely specifiable *transition functions*.

We have carried out this matching through $O(v^4)$, where v is the binary's initial orbital velocity, and through overall quadrupole order in the tidal fields [which necessitates matching though octupole order through $O(v^2)$]. There are also versions of the data that include some even higher-order terms. The data's constraint violations decrease at least as fast with increasing initial separation, b, as the uncontrolled remainders say they should (viz., $b^{-5/2}$), and often decrease much faster (b^{-n} with $n \gtrsim 5$).

The Maple scripts and C code that generate the data are freely available, either from EPAPS (see the paper), or from W. Tichy's website `<http://www.physics.fau.edu/~wolf>`. (N. Yunes has recently made some significant speed improvements to the code. These are not yet available for download, but can be obtained by contacting N. Yunes `<nyunes@space.mit.edu>` or W. Tichy `<wolf@physics.fau.edu>`.)

4. Evolutions of our data

The higher-order matched data described above are currently being evolved in two different implementations:

George Reifenberger and Wolfgang Tichy are evolving the data as is, *à la* Kelly *et al.* [11]—i.e., not solving the constraints and evolving the data constructed in the paper directly using BAM.

Tony Chu is investigating using our tidally deformed black hole metrics in superposed data *à la* Lovelace [20]—i.e., superposing the metrics using attenuation functions and then running the resulting data through a constraint solver (with appropriate excision boundary conditions) before evolving using SpEC. Here the idea is to see how much including the expected tidal deformations will improve superposed data.

5. Improving the construction

There are several ways in which one could improve the data.

Physically motivated transition functions: The transition functions are a crucial part of the construction, and are poorly constrained by formal requirements (e.g., the Frankenstein theorems [21]); the functions given in the paper are merely workable. Improved transition functions would better respect the binary's geometry (e.g., by being functions of its Newtonian potential), and might be selected from an appropriate family by minimizing the constraint violations (as suggested by J. Pullin).

Further resummations: It might be preferable to resum the black hole contributions to the PN metric using boosted, instead of unboosted Schwarzschild metrics. One could also resum parts of the multipole expansion in the far zone and the perturbative treatment of retardation in the near zone using the Liénard-Wiechert (1st post-Minkowskian [1PM]) solution (discussed in, e.g., [22], with higher-order contributions discussed in [23]). This would be akin to the Kelly *et al.* treatment of the PN metric [10]. The 1PM metric could even be used by itself in these sorts of constructions, as a simple way of including the binary's radiation in, e.g., extended superposed data.

Horizon-penetrating near zone coordinates: One might also want to convert the PN metric near the holes to the same horizon-penetrating coordinates used for the inner zone metrics to improve the quality of the matching in practice.

Constraint solving: This could significantly improve the data, particularly if one projects onto the constraint hypersurface in a physically motivated way. (How best to do this is an open problem.)

Higher-order matching: One could use, e.g., the second-order tidally perturbed black hole metric from [24] and/or calculate higher-order PN results.

Including spin: This could be done using the perturbed Kerr metric from [25] and PN results with spin [26, 27, 28]). (Eccentricity would also be possible.)

6. Hereditary effects

Since the evolution of a system in general relativity depends on its entire past history, due to tail effects, the junk radiation will have some effect on all of the binary's subsequent evolution. We indicate how to estimate this using PN results. (We do not consider the larger effect on the binary's memory, since the memory contributions to the binary's phase evolution are instantaneous.) We also consider the effects of the absence of the binary's outgoing radiation in standard initial data.

Caveats: Only lowest-order PN. Assumes no incoming radiation.

Using the (lowest-order PN) relation between Ψ_{22}, the $l = m = 2$ mode of ψ_4, and the PN mass quadrupole I_{kl} (see [29, 30]), we obtain a lowest-order tail contribution to Ψ_{22} from the junk radiation of

$$\Psi_{22}^{\rm JT}(T_R) = -2M \int_{\tau_1}^{\tau_2} \frac{\Psi_{22}(\tau)}{(T_R - \tau)^2} d\tau. \tag{1}$$

Here we have taken the junk radiation to occur during the retarded time interval $[\tau_1, \tau_2]$, with $T_R > \tau_2$. (This expression neglects the tail contributions in the relation between Ψ_{22} and I_{kl}. Additionally, we have left off all parts of the tail integral that are actually instantaneous, including the famous 11/12 bit.)

More interesting are the effects on the binary's gravitational wave luminosity. The lowest-order contribution from the junk's tail to the luminosity is given by

$$\mathcal{L}^{\rm JT}(T_R) = \frac{M}{2\pi} \mathrm{Re} \int_{-\infty}^{T_R} r\Psi_{22}^*(t) dt \int_{\tau_1}^{\tau_2} \frac{r\Psi_{22}(\tau)}{T_R - \tau} d\tau, \tag{2}$$

where M is the binary's ADM mass. For reference, the leading-order luminosity is given by

$$\mathcal{L}^{\rm LO}(T_R) = \frac{1}{8\pi} \left| \int_{-\infty}^{T_R} r\Psi_{22}(t) dt \right|^2. \tag{3}$$

Preliminary experiments indicate that the junk's tail contribution to the averaged luminosity over the first cycle of radiation is a few percent for the waveform shown in Fig. 1. This is comparable to the 2PN contributions to the luminosity at the beginning of the simulation. The junk's direct tail contribution to the waveform is also of roughly the same magnitude as the waveform itself for a short period ($\sim 10m$).

One can use the same expressions and a PN waveform to estimate the effects of leaving off the binary's initial outgoing radiation. Preliminary experiments indicate that the contribution to the luminosity is $\sim 1\%$, and the direct contributions are roughly the same as those of the junk considered above (both for the case considered above, and an initial separation of $10m$; for a separation of $20m$, the fractional effects are a factor of a few smaller).

Future work will compare the contributions to the binary's phase (computed using PN expressions) to model waveform accuracy requirements (e.g., [1, 2]), as well as studies such as [4].

References

[1] L. Lindblom, B. J. Owen, and D. A. Brown, Phys. Rev. D **78**, 124020 (2008), arXiv:0809.3844 [gr-qc].

[2] L. Lindblom, J. G. Baker, and B. J. Owen, Phys. Rev. D **82**, 084020 (2010), arXiv:1008.1803 [gr-qc].

[3] W. Tichy and P. Marronetti, Phys. Rev. D **78**, 081501(R) (2008), arXiv:0807.2985 [gr-qc].

[4] N. Bishop, D. Pollney, and C. Reisswig, arXiv:1101.5492 [gr-qc].

[5] G. Lovelace, R. Owen, H. P. Pfeiffer, and T. Chu, Phys. Rev. D **78**, 084017 (2008), arXiv:0805.4192 [gr-qc].

[6] G. Lovelace, M. A. Scheel, and B. Szilágyi, Phys. Rev. D **83**, 024010 (2011), arXiv:1010.2777 [gr-qc].

[7] H. P. Pfeiffer *et al.*, Classical Quantum Gravity **24**, S59 (2007), gr-qc/0702106.

[8] A. Buonanno, L. E. Kidder, A. H. Mroué, H. P. Pfeiffer, and A. Taracchini, arXiv:1012.1549 [gr-qc].

[9] L. Blanchet, Living Rev. Relativity **9** (2006), `http://www.livingreviews.org/lrr-2006-4`.

[10] B. J. Kelly, W. Tichy, M. Campanelli, and B. F. Whiting, Phys. Rev. D **76**, 024008 (2007), arXiv:0704.0628 [gr-qc].

[11] B. J. Kelly, W. Tichy, Y. Zlochower, M. Campanelli, and B. F. Whiting, Classical Quantum Gravity **27**, 114005 (2010), arXiv:0912.5311 [gr-qc].

[12] B. C. Mundim, B. J. Kelly, J. Zlochower, H. Nakano, and M. Campanelli, arXiv:1012.0886 [gr-qc].

[13] K. Alvi, Phys. Rev. D **61**, 124013 (2000), gr-qc/9912113.

[14] N. Yunes, W. Tichy, B. J. Owen, and B. Brügmann, Phys. Rev. D **74**, 104011 (2006), gr-qc/0503011.

[15] N. Yunes and W. Tichy, Phys. Rev. D **74**, 064013 (2006), gr-qc/0601046.

[16] S. Detweiler, Classical Quantum Gravity **22**, S681 (2005), gr-qc/0501004.

[17] G. B. Cook and M. A. Scheel, Phys. Rev. D **56**, 4775 (1997).

[18] L. Blanchet, G. Faye, and B. Ponsot, Phys. Rev. D **58**, 124002 (1998), gr-qc/9804079.

[19] M. E. Pati and C. M. Will, Phys. Rev. D **62**, 124015 (2000), gr-qc/0007087.

[20] G. Lovelace, Classical Quantum Gravity **26**, 114002 (2009), arXiv:0812.3132 [gr-qc].

[21] N. Yunes, Classical Quantum Gravity **24**, 4313 (2007), gr-qc/0611128.

[22] S. M. Kopeikin and G. Schäfer, Phys. Rev. D **60**, 124002 (1999), gr-qc/9902030.

[23] K. S. Thorne and S. J. Kovács, Astrophys. J. **200**, 245 (1975).

[24] E. Poisson and I. Vlasov, Phys. Rev. D **81**, 024029 (2010), arXiv:0910.4311 [gr-qc].

[25] N. Yunes and J. A. González, Phys. Rev. D **73**, 024010 (2006), gr-qc/0510076.

[26] H. Tagoshi, A. Ohashi, and B. J. Owen, Phys. Rev. D **63**, 044006 (2001), gr-qc/0010014.

[27] G. Faye, L. Blanchet, and A. Buonanno, Phys. Rev. D **74**, 104033 (2006), **81**, 089901(E) (2010), gr-qc/0605139.

[28] L. Blanchet, A. Buonanno, and G. Faye, Phys. Rev. D **74**, 104034 (2006), **75**, 049903(E) (2007), **81**, 089901(E) (2010), gr-qc/0605140.

[29] L. E. Kidder, Phys. Rev. D **77**, 044016 (2008), arXiv:0710.0614 [gr-qc].

[30] L. Blanchet, G. Faye, B. R. Iyer, and S. Sinha, Classical Quantum Gravity **25**, 165003 (2008), arXiv:0802.1249 [gr-qc].

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