EXERCISE 1
Using the Gaussian elimination method to solve the following systems of linear algebraic equations
(or showing that there is no solution if it is the case):
\[(a) \begin{align*}
    x_1 + x_2 + x_3 &= 2, \\
    2x_1 + x_2 &= 1, \\
    x_1 - 2x_2 - 5x_3 &= 0.
\end{align*}\]
\[(b) \begin{align*}
    x_1 - 2x_2 + 2x_3 &= 0, \\
    2x_1 + x_2 - x_3 &= 1, \\
    2x_2 - 4x_3 &= 0.
\end{align*}\]

EXERCISE 2
Find all the eigenvalues and compute the eigenvectors associated with each eigenvalue of the following matrices:
\[(a) \quad A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad (b) \quad B = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}, \quad (c) \quad C = \begin{pmatrix} 1 & i \\ -i & -1 \end{pmatrix}, \]
\[(d) \quad D = \begin{pmatrix} 0 & \sqrt{2} & 0 \\ \sqrt{2} & 0 & \sqrt{2} \\ 0 & \sqrt{2} & 0 \end{pmatrix}.\]

EXERCISE 3
Consider the system
\[x' = Ax, \quad \text{with} \quad A := \begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}\]
a) Compute the eigenvalues and eigenvectors of the matrix A.
b) Find the general solution of the above equation.

EXERCISE 4
Consider the system
\[x' = Ax, \quad \text{with} \quad A := \begin{pmatrix} -3 & -2 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}\]
a) Compute the eigenvalues and eigenvectors of the above matrix.
b) Find the general solution of the above equation.

EXERCISEs from the textbook

- Section 7.3: Complete the Problems 26 and 32. Show your work. There, \(A^T\) denotes the transpose matrix of \(A\), and \(A^*\) denotes the adjoint of \(A\) and is defined by taking the transpose and complex conjugate of \(A\). A matrix is called Hermitian or self-adjoint if \(A^* = A\).
- Section 7.3: Problem 34.
- Section: 7.4: Solve Problem 2 with \(n = 2\) (that it, do only part a,b, and c in this problem).